وزارة التعليم العالي والبحث العلمي
الجامعة التقنية الشمالية
المعهد التقني الدور

الحقيبة التعليمية

القسم العلمي:	**التقنيات الالكترونية**
اسم المقرر:	**دوائر التيار الكهربائي المتناوب**
المرحلة / المستوى:	**الاول**
الفصل الدراسي:	**الثاني**
السنة الدراسية:	**2025-2024**



اسم المقرر:	**دوائر التيار الكهربائي المتناوب**						
القسم:	التقنيات الالكترونية						
الكلية:	المعهد التقني الدور						
المرحلة / المستوى	الاول						
الفصل الدراسي:	الثاني						
عدد الساعات الاسبوعية:	نظري	2	عملي	2			
عدد الوحدات الدراسية:	4						
الرمز:	**EOTO106**						
نوع المادة	نظري		عملي		كلهما	*	
هل يتوفر نظير للمقرر في الاقسام الاخرى							
اسم المقرر النظير							
القسم							
رمز المقرر النظير							

معلومات تدريسي المادة

اسم مدرس (مدرسي) المقرر:	**فلاح محمد عبد**
اللقب العلمي:	**مدرس**
سنة الحصول على اللقب	2022
الشهادة :	**دكتوراه**
سنة الحصول على الشهادة	**2022**
عدد سنوات الخبرة (تدريس)	**20 سنة**

نموذج وصف المقرر

دراسة مفهوم الكهرباء والجهد الكهربائي والمواد العازلة والتيار ا المتناوب وكيفية ربط الدائرة الكهربائية

1	المؤسسة التعليمية	الجامعة التقنية الشمالية/المعهد التقني الدور
2	القسم العلمي / المركز	التقنيات الألكترونية
3	اسم و رمز المقرر	**دوائر التيار الكهربائي المتناوب/EOTO106**
4	اشكال الحضور المتاحة	الزامي
5	الفصل / السنة	**الفصل الثاني)15 اسبوع(/ المستوى الاول**
6	عدد الساعات الدراسية الكلي	4 ساعة أسبوعيا -)60(ساعه بالفصل
7	تأريخ اعداد الوصف	2024/2/3
8	اهداف المقرر	قدرة الطالب على ربط الدوائر الكهربائية علمياً في المختبر وتحديد الخطأ في ربط الدوائر الكهربائية.
9	مخرجات البرنامج المطلوبة وطرائق التعليم والتعلم والتقييم	
	أ- الاهداف المعرفية	
أ-1	دراسة مفهوم الكهرباء والجهد الكهربائي والمواد العازلة والتيار المستمر وكيفية ربط الدائرة الكهربائية	
	ب- الاهداف المهاراتية الخاصة بالبرنامج	
ب-1	الطريقة التقليدية في القاء المحاضرة	
ب-2	ل المواضيع)السبورة الذكية -DATA SHOW) واستخدام الأجهزة المختبرية الحديثة	
	طرائق التعليم والتعلم))المحاضرات النظرية / المحاضرات العملية / الورشة / حل الأمثلة / مشروع التخرج / التدريب الصيفي((	
	طرائق التقييم))الاختبارات الشفهية/ الاختبارات التحريرية/ التقارير الأسبوعية / الحضور اليومي/ الامتحانات الفصلية والنهائية((	
	ج- الاهداف الوجدانية والقيمية	
ج-1	قدرة الطالب على ربط الدوائر الكهربائية علمياً في المختبر	
ج-2	تنمية قدرة الطالب على تحديد الخطأ في ربط الدوائر الكهربائية	
	طرائق التعليم والتعلم))المحاضرات النظرية / المحاضرات العملية / الورشة / حل الامثلة / مشروع التخرج / التدريب الصيفي ((	
	طرائق التقييم))الاختبارات الشفهية/ الاختبارات التحريرية/ الملاحظة/ السجل التراكمي للطالب((	
	د- المهارات العامة والتأهيلية المنقولة)المهارات الأخرى المتعلقة بقابلية التوظيف والتطور الشخصي(	
د-1	تحسين مهاراتهم النقاشية	
د-2	رفع مدركاتهم البحثية ونقل الطالب من مرحلة التعليم الى التعلم	

10. المقرر

المقرر الدراسي :- دوائر التيار الكهربائي المتناوب المستوى الاول :- الفصل الثاني

الاسبوع	الساعات	مخرجات التعلم المطلوبة	اسم الموضوع	طريقة التعلم	طريقة التقييم
1	4	المعرفة والتطبيق العملي	دوائر الرنين المتسلسلة والمتوازية - حساب الجهد والتيار والمقاومة وزاوية الطور والتردد عند الرنين مع أمثلة	عملي + نظري	الاختبارات والتقارير
2	4	المعرفة والتطبيق العملي	تطبيقات لنظريات تيفينن ونورتون والعشاء مع أمثلة	عملي + نظري	الاختبارات والتقارير
3	4	المعرفة والتطبيق العملي	حساب الطاقة في دارة التيار المتردد فقط دائرة المحاثة فقط دائرة مكثف الدائرة - المقاومة ، المحاثة والمكثف في سلسلة والقوة المتوازية النشطة والمتفاعلة	عملي + نظري	الاختبارات والتقارير
4	4	المعرفة والتطبيق العملي	رسم مثلث الطاقة- تصحيح Appearnt power- عامل الطاقة	عملي + نظري	الاختبارات والتقارير
5	4	المعرفة والتطبيق العملي	ألعلى. نقل الطاقة في دائرة التيار المتردد- مع أمثلة	عملي + نظري	الاختبارات والتقارير
6	4	المعرفة والتطبيق العملي	عدد - Nodel تحليل الشبكات باستخدام تحليل معادلات العقدة	عملي + نظري	الاختبارات والتقارير
7	4	المعرفة والتطبيق العملي	Nodel أمثلة على تحليل الشبكات باستخدام تحليل	عملي + نظري	الاختبارات والتقارير
8	4	المعرفة والتطبيق العملي	دوائر التيار المتردد ثالثية الطور - توليد أحادي الطور و 2 طور وثالث مراحل لخط طاقة دلتا متصل بالطور - أمثلة للطاقة الكلية	عملي + نظري	الاختبارات والتقارير
9	4	المعرفة والتطبيق العملي	أمثلة على دوائر التيار المتناوب ثالثية الطور مع اتحادات دلتا النجمية	عملي + نظري	الاختبارات والتقارير
10	4	المعرفة والتطبيق العملي	طرق قياس القدرة لألحمال ثالثية الطور - الواط متر - اثنان واط - ثالثة واط	عملي + نظري	الاختبارات والتقارير
11	4	المعرفة والتطبيق العملي	-RL-RC - عابر DC -حالات عابرة في الدوائر عابر RLC	عملي + نظري	الاختبارات والتقارير
12	4	المعرفة والتطبيق العملي	تيارات التيار المتناوب العابرة - التيارات الجيبية RL-RC-RLC العابرة في دوائر	عملي + نظري	الاختبارات والتقارير
13	4	المعرفة والتطبيق العملي	الحث الذاتي للملف - معادلة الحث الذاتي - الحث :المتبادل بين اثنين من القولون Progresive اتصال سلسلة - توصيل سلسلة عكسية -	عملي + نظري	الاختبارات والتقارير
14	4	المعرفة والتطبيق العملي	المحولات - الهيكل - الرسم - الخواص - عملياتها وعلاقاتها - أنواع أمثلتها	عملي + نظري	الاختبارات والتقارير
15	4	المعرفة والتطبيق العملي	منحنيات التيار في دائرة الحث - الرسم الحالي وحساب الشحنة الثابتة للوقت ، تفريغ المكثفات - .التأثير الثابت للوقت - أمثلة	عملي + نظري	الاختبارات والتقارير

11	البنى التحتية	
*	الكتب المقررة المطلوبة	متوفرة في مجانية القسم ومكتبة المعهد
*	المراجع الرئيسية)الرئيسية(	متوفرة في مجانية القسم ومكتبة المعهد
	المراجع الإلكترونية، مواقع الأنترنيت	شبكة الأنترنيت

12	خطة تطوير المناهج
- استحداث مناهج دراسية ملائمة مع سوق العمل - عقد ندوات ومؤتمرات علمية تستهدف تحديث المناهج الدراسية - متابعة التطورات العلمية في مجال التخصص	

المحتوى العلمي

AC Circuits

Electronics engineering involves the use of different circuits each of which is used for various purposes. One such type of circuit is an alternating current circuit, which forms an integral part of the communication system. In this article, we will study what is an AC circuit and what is the categorization of AC circuits.
We will also study the principle of operation of AC circuits and the components that make up these circuits. We will look at some real-life applications of AC circuits to understand their advantages and disadvantages. We will emphasize the difference between AC and DC circuits as well. The article will conclude with some frequently asked questions.

AC Circuits

We know that current flows in one direction in DC circuits, but do you know that it is possible to alternate the current direction periodically? AC circuits are the type of circuits used for generating an electric current that changes its direction and magnitude periodically. Here is a formal definition of AC circuits
AC circuits are powered by alternating sources and produce current and voltage that vary in magnitude and direction over time, after which they repeat the cycle.

Important Terminologies

Let us know about some Terminologies of AC circuits.

- **Amplitude:** The maximum peak value that the voltage or current can take in one full cycle is known as the amplitude of the alternating quantity.
- **Frequency:** Frequency is the number of cycles or altercations a quantity can make in one second. It is usually measured in Hertz.
- **Time period:** The time taken by an alternating quantity like current or voltage to complete one oscillation or cycle is known as the period of that quantity.
- **Instantaneous Value:** The value of any quantity at that particular instant of time is known as the instantaneous value of that quantity.
- **Waveform:** Every alternating quantity when measured over a period of time creates a shape known as waveform of that quantity. This waveform is generally plotted on y axis with respect to time.

AC Circuit Formulas

Here is a list of important formulas

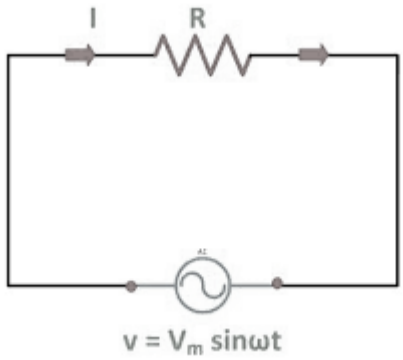
Quantity	Formula
AC voltage	$v=V_0 \sin \omega t$
AC current	$i=I_0 \sin \omega t$
Capacitive Reactance	$X_c=1/2\pi fC$
RMS voltage	$V_{rms}=V_0/\sqrt{2}$
rms current	$I_{rms}=I_0/\sqrt{2}$
Inductive reactance	$X_L=2\pi fL$
The phase angle of an RLC series circuit	$\phi=\tan^{-1}[(X_L-X_C)/R]$
AC version of Ohm's law	$I_o= V_o/R$
Impedance of an RLC series circuit	$Z= \sqrt{(R^2+(X_L-X_C)^2}$
Average power dissipated by a resistor	$P_{avg}= I_{rms}^2R$

Types of AC Circuit

There are different types of AC circuits. Let us take a look at them.

Purely Resistive

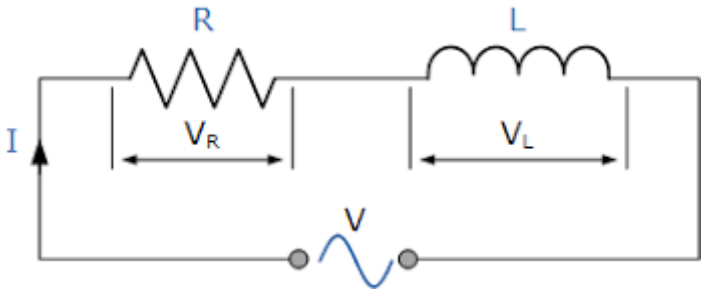
This circuit is categorized by low inductance which reduces the overall reactance in comparison to the resistance of the circuit. The resistance is then used to regulate the current flowing in the circuit. Since the inductance is almost negligible, we can call it a non-inductive circuit whose frequency doesn't change with changing frequency. This is what the purely resistive circuit looks like.



$V=V_m sin(\omega t) \Rightarrow I=V/R \Rightarrow I=V_m sin(\omega t)/R$

Resistor-Inductor Circuit

A resistor-inductor circuit is a type of circuit that has a resistor R and a pure inductor of value L. These can be connected in series with an AC power source. In this circuit, a magnetic field is established in the inductor due to the current flowing in the circuit. The current is periodic and has a time constant known as the inductive time constant. Let us take a look at the Resistive-inductive circuit.

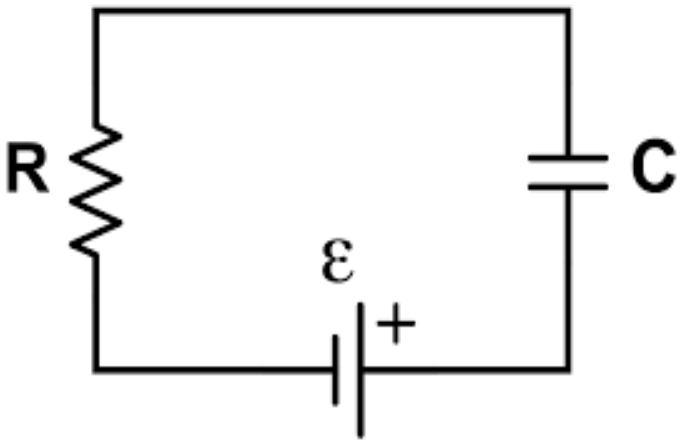


Resistive-inductive circuit

$I= V/R$ *where* $V=V_m sin(\omega t)$ *and* $R= \sqrt{R^2+X_{L2}}$ *where* $X_L=j\omega L$

Resistor-Capacitor Circuit

A resistor-capacitor circuit is a type of circuit that has a resistor R and a pure capacitor of value C. These can be connected in series with an AC power source. In this circuit, an electric field is established in the capacitor due to the charge collected in the capacitor. The current grows or decays exponentially and has a time constant known as the capacitive time constant. Let us take a look at the Resistive-capacitive circuit.

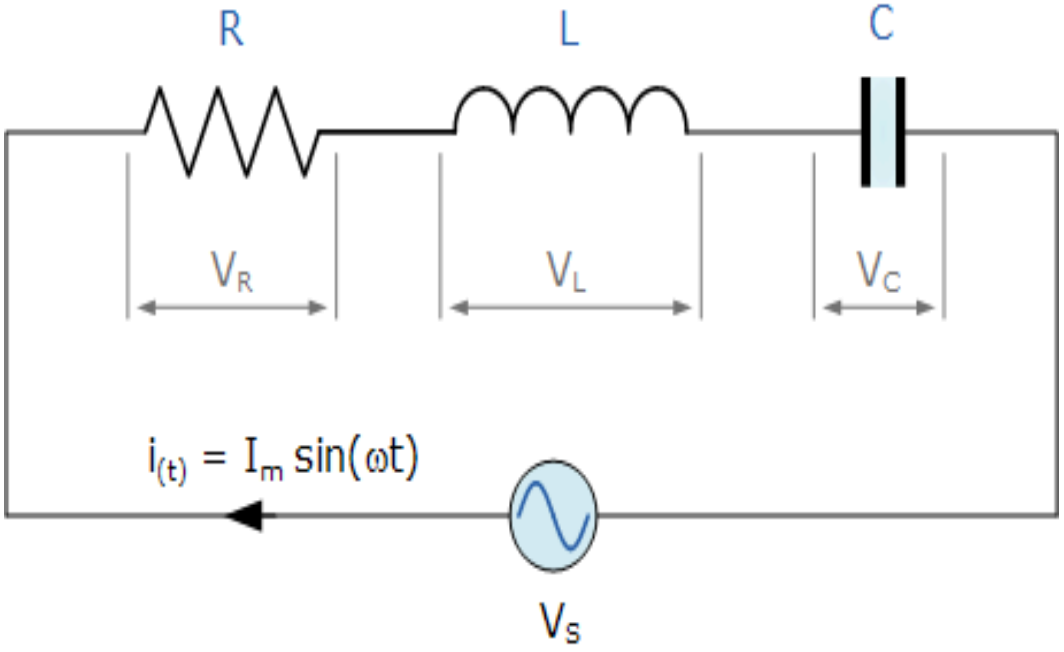


Resistive Capacitor circuit

$I= V/R$ where $V=V_m\sin(\omega t)$ and $R= \sqrt{R^2+X_c^2}$ where $X_c=1/j\omega C$

Resistor Inductor Capacitor circuit

An RLC circuit consists of a resistor, inductor, and a capacitor connected in a series of parallel. The impedance of the inductor and capacitor are frequency dependent, therefore, the output of the series RLC circuit varies with a change in the frequency of the circuit. Therefore, the voltage and current in this circuit exist in phases. Let us take a look at the RLC circuit and the impedance of such a circuit.



RLC Circuit

$I= V/R$ where $V=V_m\sin(\omega t)$ and $R= \sqrt{R^2+(X_L-X_c)^2}$ where $X_L=j\omega L$ and $X_c=1/j\omega C$

Working principle

The principle of working AC circuits is the rotation of a magnet along a set of wires or coils under the effect of the constant magnetic field. An equivalent

version of this principle can be understood as if any coil is rotated in a stationary consistent magnetic field, then a current is generated in an AC generator which alternates periodically. Note that this generator is known as an alternator. This principle is applied to designing AC circuits.

AC Circuit Analysis

The term analysis is used for understanding the circuit and calculating the current and voltage components of the circuit. Since we are talking about the AC circuits, the current and voltage values calculated will vary periodically with frequency, and on plotting these values on y axis with time, we will get a sinusoidal function. The voltage generated in the circuit can be calculated by measuring the peak or the rms voltage of the circuit. This voltage can then be used for calculating the current in the circuit if you know the circuit impedance.

AC Circuit Theory

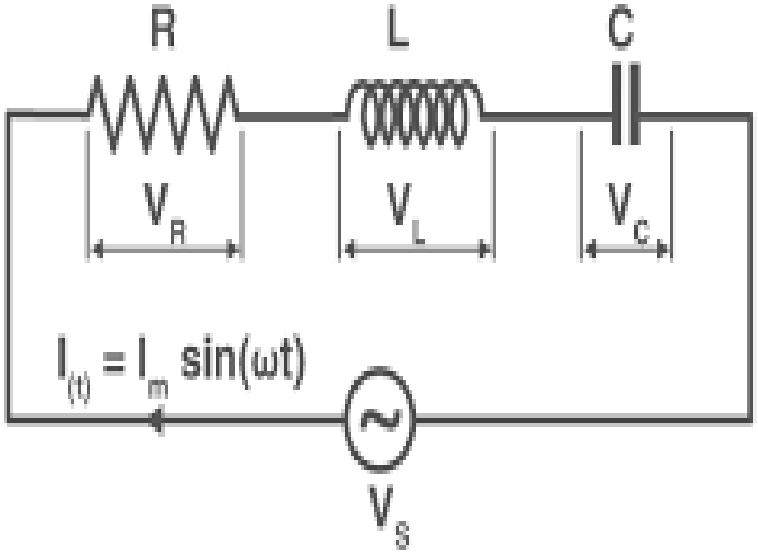
We have seen the principle of AC circuits is the rotation of a magnet along a set of wires or coils under the effect of the constant magnetic field. Let us formally define AC Circuit Theory

The alternating current theory talks about the steady-state behavior of electrical circuits and studies the fluctuations in current and voltage with time.

Fourier series are used for analyzing these alternating signals in the shape of a sinusoid because every signal can be represented using sinusoids.

Construction

Let us take a look at the construction of AC circuits.



AC circuit

As we can see, the given circuit is made up of passive components like a resistor, capacitor and an inductor and it also consists of an alternating power source. Let us study these components carefully.

Components

These are the components of AC circuits

Resistor: Resistors are used to limit or regulate the amount of current flowing in the circuit. It is the passive component of the circuit that dissipates heat. The resistance is independent of frequency and can be written as
Resistance= R

Capacitor: A capacitor is a passive device with two terminals that is used for storing the electrical energy generated in the circuit in the form of charges. The impedance of the Capacitor is frequency-dependent and can be written as
$X_C=1/jwC$

where w=frequency, C=capacitance

Inductor: An inductor is also a two-terminal device that stores electrical energy in the form of a magnetic field. This stored magnetic field depends on the current flowing in the circuit. The impedance of the inductor is frequency dependence and can be written as
$X_L=jwL$ *where w=frequency, L=inductance*

Difference Between AC and DC Circuits

Let us study the difference between AC and DC circuits.

Parameter	AC circuits	DC circuits
Current	The current flowing in the circuit alternates in its direction.	Current in the DC circuit flows in one direction.
Power Supply	AC generators are used in power supply.	Batteries are used as power supply.
Frequency	The circuit has a non-zero frequency.	The circuit has 0 frequency.
Losses	Power can be transferred over long distances without much power loss.	High losses are associated so long-distance communication is not possible.
Power	Reactive power is generated.	No reactive power is generated.

Advantages of AC Circuits

Let us study some advantages of AC circuits.

- AC circuits have the ability to produce High voltages therefore High power with full efficiency.
- The cost of installing an AC circuit is less as compared to DC circuits making AC a popular choice.

- The power loss in AC circuits is minimal over long distance ranges which is Contrary to DC which has huge power loss with increasing distances
- AC voltage is easier to step up and step down using a transformer. This is not possible in DC circuits.
- The conversion of AC circuits to DC circuits can be done easily depending on the need but conversion from DC to AC is difficult.

Disadvantages of AC Circuits

Let us study some disadvantages of AC circuits.

- Due to the alternating nature of current in AC circuits, these circuits can't be used for charging different appliances.
- The skin effect is prominent in AC circuits which reduces the actual amount of current flowing in the circuit.
- Components of AC circuits are prone to vibrations due to their frequency dependence therefore they are short-lived.
- Voltage regulation in AC circuits is difficult due to the high voltage drop.
- Working with AC circuits is very dangerous due to the high voltage generated therefore High supervision is required with proper precautions.

Applications of AC Circuits

Let us study some applications of AC circuits.

- The power fed to households is AC in nature and uses AC circuits to ensure power is available at large distances.
- Electric motors in power plants operate on AC circuits. These motors are used to generate electrical energy from mechanical energy.
- AC circuits are used for generating power to drive electric vehicles therefore AC circuits have an important role in transportation.
- Majority of appliances like lights, fans, microwave ovens, washing machines, and kettles need alternating current to operate therefore they make use of AC circuits.
- AC circuits form a core part of the syllabus in institutions since the concepts of AC help to form the base of electronics engineering.

AC circuits: alternating current electricity

Time and phasor animations are used to explain alternating current (AC) circuits. Impedance, phase relations, resonance and RMS quantities are shown on this resource page from Physclips: a multi-level, multimedia introduction to physics (download the animations on this page).

The voltage across a resistor (upper trace) and an inductor (lower trace) in series. Note the phase difference.

AC electricity is ubiquitous not only in the supply of power, but in electronics and signal processing. Here are some of the questions we'll answer:

- If impedance is voltage over current, why is it different from resistance?
- What are RMS values?
- How can you have 100 volts across a capacitor, 100 volts across an inductor, and 10 volts across the two of them in series?
- What does it mean to say that current and voltage are out of phase?
- Why and how does the ratio of voltage to current in capacitors and inductors depend on frequency?

We introduce the voltage-current relations for resistors, capacitors and inductors separately using animations to show the time-varying nature, and why frequency is important. Then we combine the components in series and parallel.

Sub-headings on this page are:

- Resistors in AC circuits
- What are impedance and reactance?
- Capacitors and charging
- Inductors and the Farady emf
- Impedance of components
- RC Series combinations
- RL Series combinations
- RLC Series combinations
- Resonance
- Bandwidth and Q factor
- Complex Impedance
- (*more to come*)

There are separate pages on

- Ohm's law and drift velocity in conductors
- RC filters, integrators and differentiators
- LC oscillations
- Power, RMS values and three-phase circuits
- Transformers, and
- Motors and generators

But first, **why study AC circuits?** You probably live in a house or appartment with sockets that deliver AC. Your radio, television and portable phone receive it, using

(among others) circuits like those below. As for the device you're using to read this, its signals are not ordinary sinusoidal AC, but, thanks to Fourier's theorem, *any* varying signal may be analysed in terms of its sinusoidal components. So AC signals are almost everywhere. And you can't escape the components, because even the electrical circuits in your brain use capacitors and resistors.

Some terminology

For brevity, we shall refer to electrical potential difference as voltage. Throughout this page, we shall consider voltages and currents that vary sinusoidally with time. We shall use lower case v and i for the voltage and current when we are considering their variation with time explicitly. The **amplitude** or **peak value** of the sinusoidal variation we shall represent by V_m and I_m, and we shall use $V = V_m/\sqrt{2}$ and $I = I_m/\sqrt{2}$ without subscripts to refer to the RMS values. For an explanation of RMS values, see Power and RMS values. For the origin of the sinusoidally varying voltage in the mains supply, see Motors and generators.

So for instance, we shall write:

$v = v(t) = V_m \sin(\omega t + \varphi)$
$i = i(t) = I_m \sin(\omega t)$.
where ω is the **angular frequency**. $\omega = 2\pi f$, where f is the ordinary or cyclic frequency. f is the number of complete oscillations per second. φ is the **phase difference** by which the voltage leads the current. We shall meet this and the geometrical significance of ω later.

Resistors and Ohm's law in AC circuits

The voltage v across a resistor is proportional to the current i travelling through it. (See the page on drift velocity and Ohm's law.) Further, this is true at all times: v = Ri. So, if the current in a resistor is
$i = I_m . \sin(\omega t)$, we write:
$v = R.i = R.I_m \sin(\omega t)$
$v = V_m . \sin(\omega t)$ where
$V_m = R.I_m$

So for a resistor, the peak value of voltage is R times the peak value of current. Further, they are in phase: when the current is a maximum, the voltage is also a maximum. (Mathematically, $\varphi = 0$.) The first animation shows the voltage and current in a resistor as a function of time.

The rotating lines in the right hand part of the animation are a very simple case of a **phasor diagram** (named, I suppose, because it is a vector representation of phase). With respect to the x and y axes, radial vectors or phasors representing the current and the voltage across the resistance rotate with angular velocity ω. The lengths of these phasors represent the peak current I_m and voltage V_m. The y components are $I_m \sin(\omega t) = i(t)$ and voltage $V_m \sin(\omega t) = v(t)$. You can compare i(t) and v(t) in the animation with the vertical components of the phasors. The animation and phasor diagram here are simple, but they will become more useful when we consider components with different phases and with frequency dependent behaviour.

(To see where this rotating phasor idea comes from, see the comparison of simple harmonic motion and circular motion in Physclips.)

What are impedance and reactance?

Circuits in which current is proportional to voltage are called **linear circuits**. (As soon as one inserts diodes and transistors, circuits cease to be linear, but that's another story.) The ratio of voltage to current in a resistor is its **resistance**. Resistance does not depend on frequency, and in resistors the two are in phase, as we have seen above. However, circuits with only resistors are not very interesting.

In general, the ratio of voltage to current *does* depend on frequency and in general there *is* a phase difference. So **impedance** is the general name we give to the ratio of voltage to current. It has the symbol Z. Resistance is a special case of impedance. Another special case is that in which the voltage and current are out of phase by 90°: this is an important case because when this happens, no power is lost in the circuit. In this case where the voltage and current are out of phase by 90°, the ratio of voltage to current is called the **reactance**, and it has the symbol X. We return to summarise these terms and give expressions for them below in the section Impedance of components, but first let us see why there are frequency dependence and phase shifts for capacitors and for inductors.

Capacitors and charging

The voltage on a capacitor depends on the amount of charge you store on its plates. The current flowing onto the positive capacitor plate (equal to that flowing off the negative plate) is by definition the rate at which charge is being stored. So the charge Q on the capacitor equals the integral of the current with respect to time. From the definition of the capacitance,

$v_C = q/C$, so

$$v = \frac{q}{C} = \frac{1}{C}\int i.dt$$

Now remembering that the integral is the area under the curve (shaded blue), we can see in the next animation why the current and voltage are out of phase.

Once again we have a sinusoidal current $i = I_m \cdot \sin(\omega t)$, so integration gives

$$v = \frac{1}{C}\int I_m \sin(\omega t)\, dt$$

$$= -\frac{1}{\omega C} I_m \cos(\omega t)$$

$$= \frac{1}{\omega C} I_m \sin(\omega t - \pi/2)$$

$$= X_C . I_m \sin(\omega t - \pi/2)$$

(The constant of integration has been set to zero so that the average charge on the capacitor is 0).

Now we define the **capacitive reactance** X_C as the ratio of the magnitude of the voltage to magnitude of the current in a capacitor. From the equation above, we see that $X_C = 1/\omega C$. Now we can rewrite the equation above to make it look like Ohm's law. The voltage is proportional to the current, and the peak voltage and current are related by

$$V_m = X_C . I_m .$$

Note the two important differences. First, there is a difference in phase: the integral of the sinusoidal current is a negative cos function: it reaches its maximum (the capacitor has maximum charge) when the current has just finished flowing forwards and is about to start flowing backwards. Run the animation again to make this clear. Looking at the relative phase, the voltage across the capacitor is 90°, or one quarter cycle, behind the current. We can see also see how the $\varphi = 90°$ phase difference affects the phasor diagrams at right. Again, the vertical component of a phasor arrow represents the instantaneous value of its quanitity. The phasors are rotating counter clockwise (the positive direction) so the phasor representing V_C is 90° *behind* the current (90° clockwise from it).

Recall that **reactance** is the name for the ratio of voltage to current when they differ in phase by 90°. (If they are in phase, the ratio is called resistance.) Another difference between reactance and resistance is that the reactance is **frequency dependent**. From the algebra above, we see that the capacitive reactance X_C decreases with frequency . This is shown in the next animation:

when the frequency is halved but the current amplitude kept constant, the capacitor has twice as long to charge up, so it generates twice the potential difference. The blue shading shows q, the integral under the current curve (light for positive, dark for negative). The second and fourth curves show $V_C = q/C$. See how the lower frequency leads to a larger charge (bigger shaded area before changing sign) and therefore a larger V_C.

Thus for a capacitor, the ratio of voltage to current **decreases with frequency**. We shall see later how this can be used for filtering different frequencies.

Inductors and the Farady emf

An inductor is usually a coil of wire. In an ideal inductor, the resistance of this wire is negligibile, as is its capacitance. The voltage that appears across an inductor is due to its own magnetic field and Faraday's law of electromagnetic induction. The current i(t) in the coil sets up a magnetic field, whose magnetic flux φ_B is proportional to the field strength, which is proportional to the current flowing. (Do not confuse the phase φ with the flux φ_B.) So we define the (self) inductance of the coil thus:

$$\varphi_B(t) = L.i(t)$$

Faraday's law gives the emf $E_L = - d\varphi_B/dt$. Now this emf is a voltage rise, so for the voltage drop v_L across the inductor, we have:

$$\begin{aligned} v_L(t) = -E_L &= \frac{d\phi_B}{dt} = \frac{d}{dt}(Li) \\ &= L\frac{d}{dt} I_m \sin(\omega t) \\ &= \omega L I_m \cos(\omega t) \\ &= \omega L I_m \sin(\omega t + \pi/2) \\ &= X_L I_m \sin(\omega t + \pi/2) \end{aligned}$$

Again we define the **inductive reactance** X_L as the ratio of the magnitudes of the voltage and current, and from the equation above we see that $X_L = \omega L$. Again we note the analogy to Ohm's law: the voltage is proportional to the current, and the peak voltage and currents are related by

$$V_m = X_L.I_m.$$

Remembering that the derivative is the local slope of the curve (the purple line), we can see in the next animation why voltage and current are out of phase in an inductor.

Again, there is a difference in phase: the derivative of the sinusoidal current is a cos function: it has its maximum (largest voltage across the inductor) when the current is changing most rapidly, which is when the current is intantaneously zero. The animation should make this clear. The voltage across the ideal inductor is 90° ahead of the current, (ie it reaches its peak one quarter cycle before the current does). Note how this is represented on the phasor diagram.

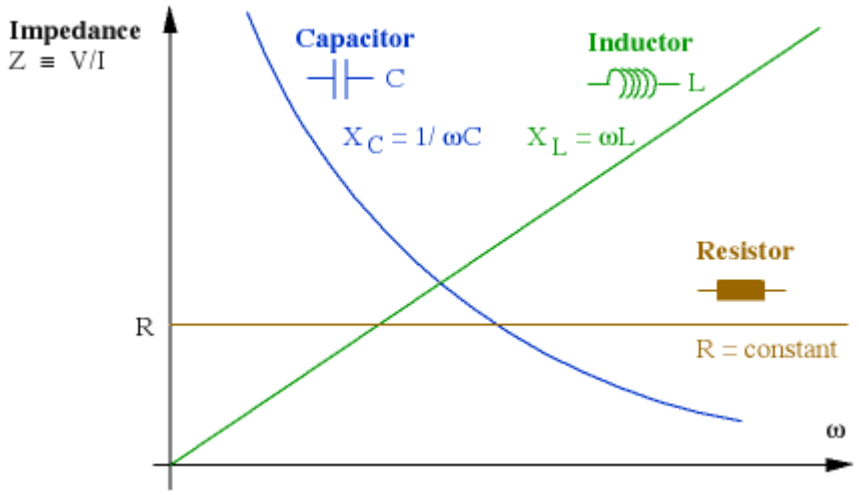
Again we note that the reactance is **frequency dependent**: $X_L = \omega L$. This is shown in the next animation: when the frequency is halved but the current amplitude kept constant, the current is varying only half as quickly, so its derivative is half as great, as is the Faraday emf. For an inductor, the ratio of voltage to current increases with frequency, as the next animation shows.

Impedance of components

Let's recap what we now know about voltage and curent in linear components. The **impedance** is the general term for the ratio of voltage to current. Resistance is the special case of impedance when $\varphi = 0$, reactance the special case when $\varphi = \pm 90°$. The table below summarises the impedance of the different components. It is easy to remember that the voltage on the capacitor is *behind* the current, because the charge doesn't build up until after the current has been flowing for a while.

Resistor 	**Capacitor** 	**Inductor** 
Resistance	Capacitive reactance	Inductive reactance
$V_R/I = R$	$V_C/I = X_C = \frac{1}{\omega C}$	$V_L/I = X_L = \omega L$
V and I in phase	V lags I by $\pi/2$	V leads I by $\pi/2$

The same information is given graphically below. It is easy to remember the frequency dependence by thinking of the DC (zero frequency) behaviour: at DC, an inductance is a short circuit (a piece of wire) so its impedance is zero. At DC, a capacitor is an open circuit, as its circuit diagram shows, so its impedance goes to infinity.



RC Series combinations

When we connect components together, Kirchoff's laws apply at any instant. So the voltage v(t) across a **resistor and capacitor** in series is just

$v_{series}(t) = v_R(t) + v_C(t)$

However, the addition is complicated because the two are not in phase. The next animation makes this clear: they add to give a new sinusoidal voltage, but the amplitude is *less* than $V_{mR}(t) + V_{mC}(t)$. Similarly, the AC voltages ($V = V_m/\sqrt{2}$) do not add up. This may seem confusing, so it's worth repeating:

$v_{series} = v_R + v_C$ but

$V_{series} < V_R + V_C$.

This should be clear on the animation and the still graphic below: check that the voltages v(t) do add up, and then look at the magnitudes. The amplitudes and the RMS voltages V do not add up in a simple arithmetical way. Instead, they add like *vectors*.

Here's where *phasor diagrams* are going to save us a lot of work. Play the animation again (click play), and look at the projections on the vertical axis. Because we have sinusoidal variation in time, the vertical component (magnitude times the sine of the angle it makes with the x axis) gives us v(t). But the y components of different vectors, and therefore phasors, add up simply. Using bold for vectors or phasors: if

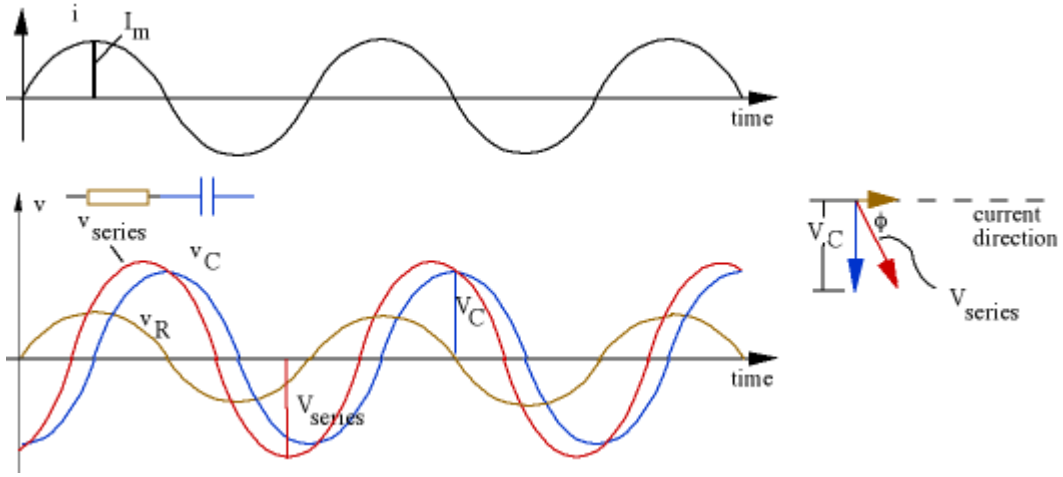
$\mathbf{r}_{total} = \mathbf{r}_1 + \mathbf{r}_2$, then

$r_{y\,total} = r_{y1} + r_{y2}$.

So v(t), the sum of the y projections of the component phasors, is just the y projection of the sum of the component phasors. So we can represent the three sinusoidal voltages by their phasors. (While you're looking at it, check the phases. You'll see that the series voltage is behind the current in phase, but the

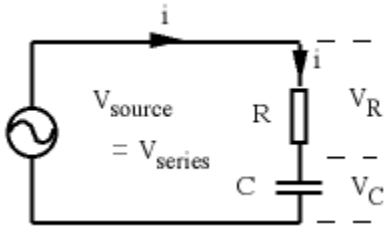
relative phase is somewhere between 0 and 90°, the exact value depending on the size of V_R and V_C. We'll discuss phase below.)

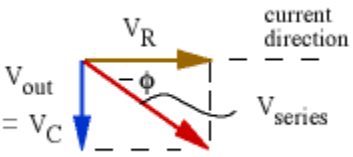
Now let's stop that animation and label the values, which we do in the still figure below. All of the variables (i, v_R, v_C, v_{series}) have the same frequency f and the same angular frequency ω, so their phasors rotate together, with the same relative phases. So we can 'freeze' it in time at any instant to do the analysis. The convention I use is that the x axis is the reference direction, and the reference is whatever is common in the circuit. In this series circuit, the current is common. (In a completely parallel circuit, the voltage is common, so I would make the voltage the horizontal axis.) Be careful to distinguish v and V in this figure!



(Careful readers will note that I'm taking a shortcut in these diagrams: the size of the arrows on the phasor diagrams are drawn the same as the amplitudes on the v(t) graphs. However I am just calling them V_R, V_C etc, rather than V_{mR}, V_{mR} etc. The reason is that the peak values (V_{mR} etc) are rarely used in talking about AC: we use the RMS values, which are peak values times 0.71. Phasor diagrams in RMS have the same shape as those drawn using amplitudes, but everything is scaled by a factor of $0.71 = 1/\sqrt{2}$.)

The phasor diagram at right shows us a simple way to calculate the series voltage. The components are in series, so the current is the same in both. The voltage phasors (brown for resistor, blue for capacitor in the convention we've been using) add according to vector or phasor addition, to give the series voltage (the red arrow). By now you don't need to look at v(t), you can go straight from the circuit diagram to the phasor diagram, like this:





From Pythagoras' theorem:

$$V^2_{mRC} = V^2_{mR} + V^2_{mC}$$

If we divide this equation by two, and remembering that the RMS value V = $V_m/\sqrt{2}$, we also get:

$$V_{RC}^2 = V_R^2 + V_C^2$$

so, using the resistance and the capacitive reactance $1/\omega C$:

$$V_{RC}^2 = (IR)^2 + \left(\frac{I}{\omega C}\right)^2$$

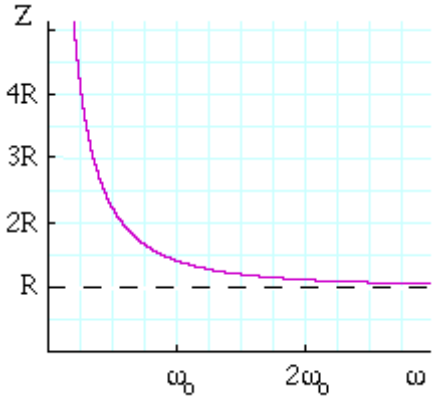
so :

$$V_{RC} = \sqrt{(IR)^2 + \left(\frac{I}{\omega C}\right)^2}$$

$$= I\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

Now this looks like Ohm's law again: V is proportional to I. Their ratio is the **series impedance**, Z_{series} and so for this series circuit,

$$Z_{RC} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$



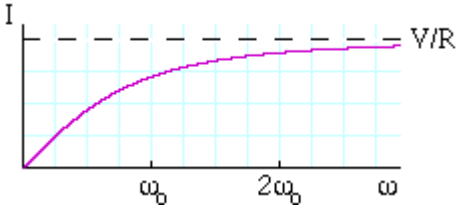
Note the frequency dependence of the series impedance Z_{RC}: at low frequencies, the impedance is very large, because the capacitive reactance $1/\omega C$ is large (the capacitor is open circuit for DC). At high frequencies, the capacitive reactance goes to zero (the capacitor doesn't have time to charge up) so the series impedance goes to R. At the angular frequency $\omega = \omega_o = 1/RC$, the capacitive reactance $1/\omega C$ equals the resistance R. We shall show this characteristic frequency on all graphs on this page.

Remember how, for two resistors in series, you could just add the resistances: $R_{series} = R_1 + R_2$ to get the resistance of the series combination. That simple result comes about because the two voltages are both in phase with the current, so their phasors are parallel. Because the phasors for reactances are 90° out of phase with the current, the series impedance of a resistor R and a reactance X are given by Pythagoras' law:

$Z_{series}^2 = R^2 + X^2$.

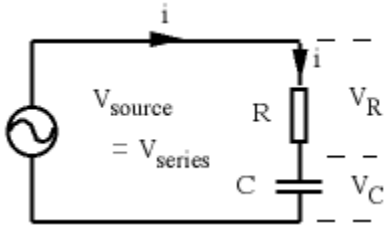
Ohm's law in AC. We can rearrange the equations above to obtain the current flowing in this circuit. Alternatively we can simply use the Ohm's Law analogy and say that $I = V_{source}/Z_{RC}$. Either way we get

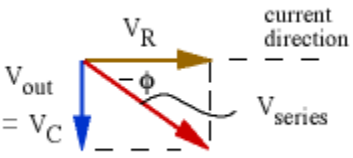
$$I = \frac{V_{source}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$



where the current goes to zero at DC (capacitor is open circuit) and to V/R at high frequencies (no time to charge the capacitor).

So far we have concentrated on the magnitude of the voltage and current. We now derive expressions for their relative phase, so let's look at the phasor diagram again.



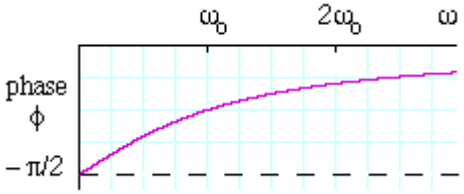


From simple trigonometry, the angle by which the current leads the voltage is

$\tan^{-1}(V_C/V_R) = \tan^{-1}(IX_C/IR)$

$= \tan^{-1}(1/\omega RC) = \tan^{-1}(1/2\pi fRC)$.

However, we shall refer to the angle φ by which the voltage leads the current. The voltage is *behind* the current because the capacitor takes time to charge up, so φ is negative, ie



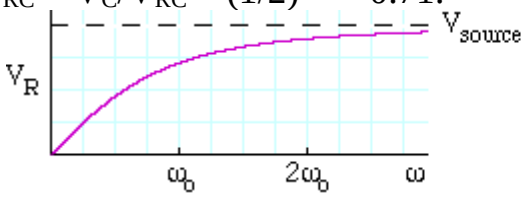
$\varphi = -\tan^{-1}(1/\omega RC) = \tan^{-1}(1/2\pi fRC)$.

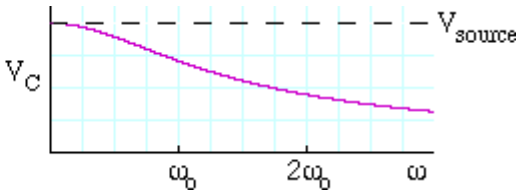
(You may want to go back to the RC animation to check out the phases in time.)

At low frequencies, the impedance of the series RC circuit is dominated by the capacitor, so the voltage is 90° behind the current. At high frequencies, the impedance approaches R and the phase difference approaches zero. The frequency dependence of Z and φ are important in the applications of RC circuits. The voltage is mainly across the capacitor at low frequencies, and mainly across the resistor at high frequencies. Of course the two voltages must add up to give the voltage of the source, but they add up as vectors and, from Pythagoras:

$V^2_{RC} = V^2_R + V^2_C$.

At the frequency $\omega = \omega_o = 1/RC$, the phase $\varphi = 45°$ and the voltage fractions are $V_R/V_{RC} = V_C/V_{RC} = (1/2)^{1/2} = 0.71$.

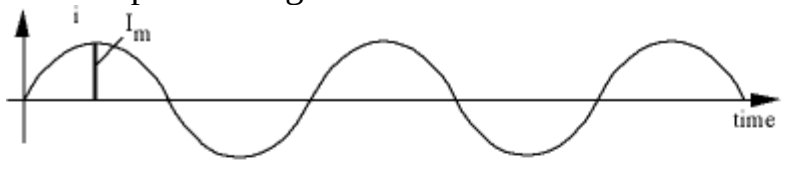


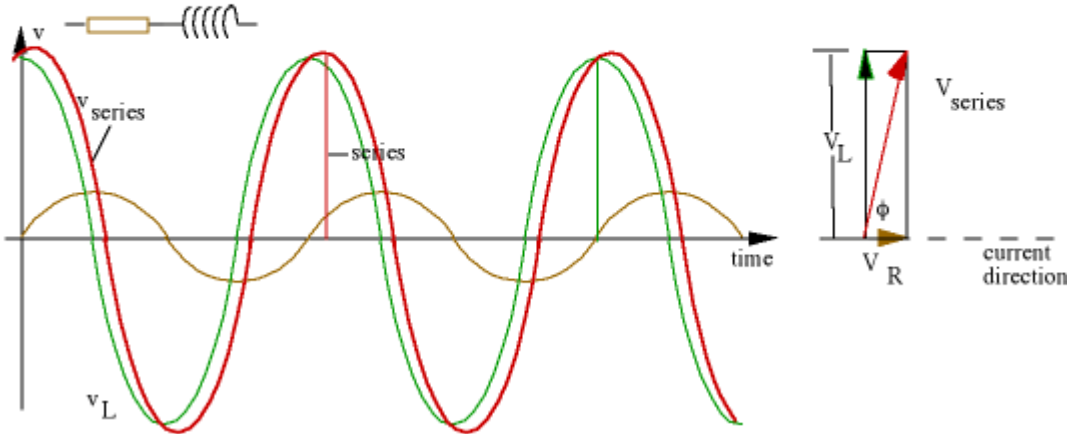


So, by chosing to look at the voltage across the resistor, you select mainly the high frequencies, across the capacitor, you select low frequencies. This brings us to one of the very important applications of RC circuits, and one which merits its own page: filters, integrators and differentiators where we use sound files as examples of RC filtering.

RL Series combinations

In an RL series circuit, the voltage across the inductor is *ahead*of the current by 90°, and the inductive reactance, as we saw before, is $X_L = \omega L$. The resulting v(t) plots and phasor diagram look like this.





It is straightforward to use Pythagoras' law to obtain the series impedance and trigonometry to obtain the phase. We shall not, however, spend much time on RL circuits, for three reasons. First, it makes a good exercise for you to do it yourself. Second, RL circuits are used much less than RC circuits. This is because inductors are always* too big, too expensive and the wrong value, a claim you can check by looking at an electronics catalogue. If you can use a circuit involving any number of Rs, Cs, transistors, integrated circuits etc to replace an inductor, one usually does. The third reason why we don't look

closely at RL circuits on this site is that you can simply look at RLC circuits (below) and omit the phasors and terms for the capacitance.

* Exceptions occur at high frequencies (~GHz) where only small value Ls are required to get substantial ωL. In such circuits, you make an inductor by twisting copper wire around a pencil and adjusts its value by squeezing it with your fingers.

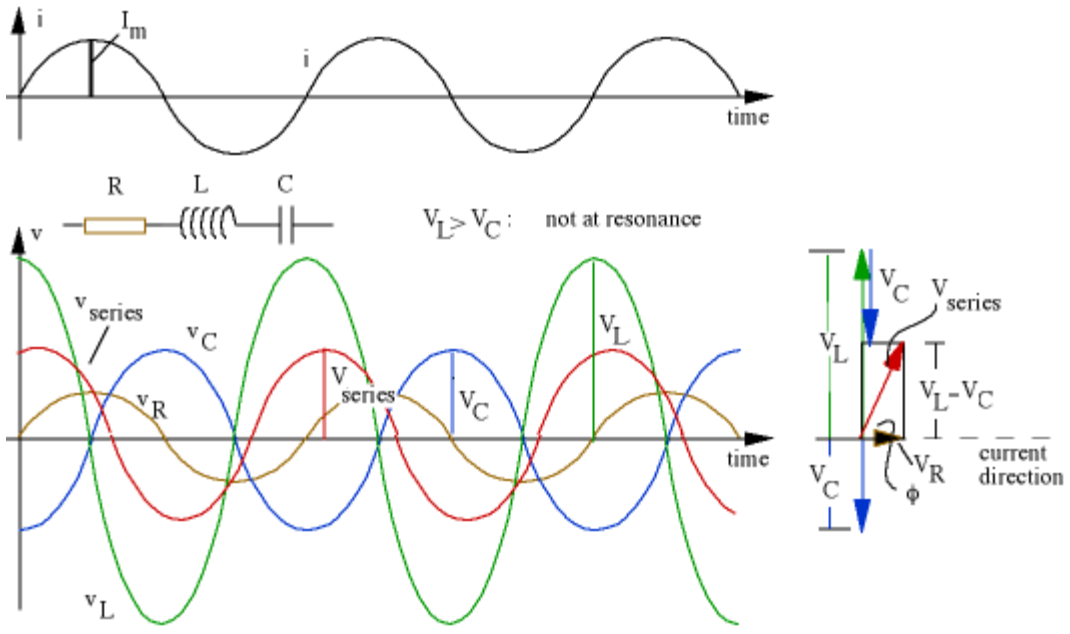
RLC Series combinations

Now let's put a **resistor, capacitor and inductor in series**. At any given time, the voltage across the three components in series, $v_{series}(t)$, is the sum of these:

$$v_{series}(t) = v_R(t) + v_L(t) + v_C(t),$$

The current i(t) we shall keep sinusoidal, as before. The voltage across the resistor, $v_R(t)$, is in phase with the current. That across the inductor, $v_L(t)$, is 90° ahead of the current and that across the capacitor, $v_C(t)$, is 90° behind.

Once again, the time-dependent voltages v(t) add up at any time, but the RMS voltages V do not simply add up. Once again they can be added by *phasors* representing the three sinusoidal voltages. Again, let's 'freeze' it in time for the purposes of the addition, which we do in the graphic below. Once more, be careful to distinguish v and V.



Look at the phasor diagram: The voltage across the ideal inductor is antiparallel to that of the capacitor, so the total reactive voltage (the voltage which is 90° ahead of the current) is $V_L - V_C$, so Pythagoras now gives us:

$$V^2_{series} = V^2_R + (V_L - V_C)^2$$

Now $V_R = IR$, $V_L = IX_L = I\omega L$ and $V_C = IX_C = I/\omega C$. Substituting and taking the common factor I gives:

$$V = I\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = I Z_{series}$$

where Z_{series} is the **series impedance**: the ratio of the voltage to current in an RLC series ciruit. Note that, once again, reactances and resistances add according to Pythagoras' law:

$$Z_{series}^2 = R^2 + X_{total}^2$$
$$= R^2 + (X_L - X_C)^2.$$

Remember that the inductive and capacitive phasors are 180° out of phase, so their reactances tend to cancel.

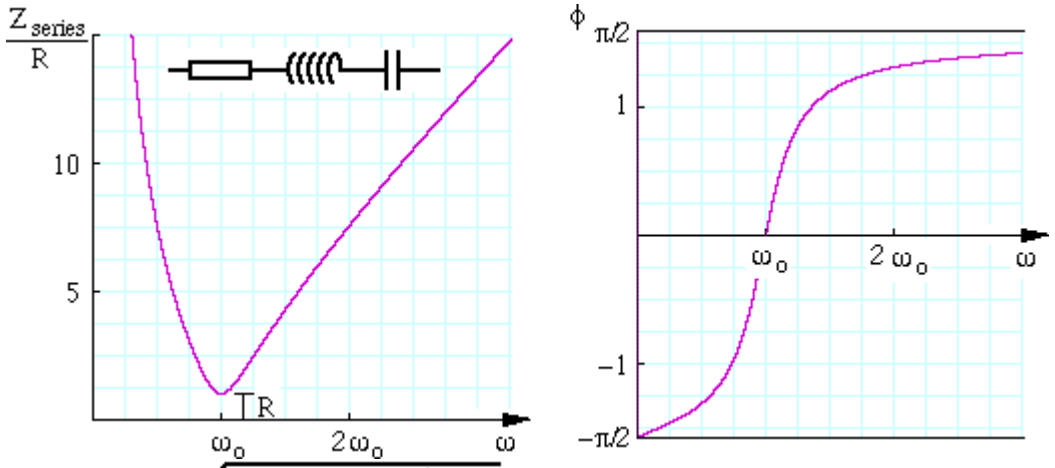
Now let's look at the **relative phase**. The angle by which the voltage leads the current is

$$\varphi = \tan^{-1}((V_L - V_C)/V_R).$$

Substituting $V_R = IR$, $V_L = IX_L = I\omega L$ and $V_C = IX_C = I/\omega C$ gives:

$$\phi = \tan^{-1}\frac{\omega L - 1/\omega C}{R}$$

The dependence of Z_{series} and φ on the angular frequency ω is shown in the next figure. The angular frequency ω is given in terms of a particular value ω_o, the resonant frequency ($\omega_o^2 = 1/LC$), which we meet below.



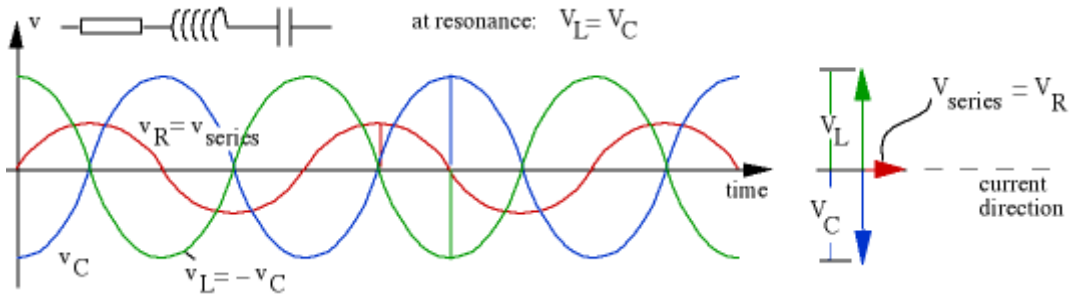
$$Z_{series} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\phi = \tan^{-1}\frac{\omega L - 1/\omega C}{R}$$

(Setting the inductance term to zero gives back the equations we had above for RC circuits, though note that phase is negative, meaning (as we saw above) that

voltage lags the current. Similarly, removing the capacitance terms gives the expressions that apply to RL circuits.)

The next graph shows us the special case where the frequency is such that $V_L = V_C$.



Because $v_L(t)$ and v_C are 180° out of phase, equal amplitude means that $v_L(t) = - v_C(t)$, so the **two reactive voltages cancel out**, and the series voltage is just equal to that across the resistor. This case is called series resonance, which is our next topic.

Resonance

The graph and the expression for the series impedance show that series impedance goes to infinity at high frequency because of the presence of the inductor, which produces a large emf if the current varies rapidly. Similarly it is large at very low frequencies because of the capacitor, which has a long time in each half cycle in which to charge up. As we saw in the plot of $Z_{series}\omega$ above, there is a minimum value of the series impedance, when the voltages across capacitor and inductor are equal and opposite, ie $v_L(t) = - v_C(t)$ so $V_L(t) = V_C$, so

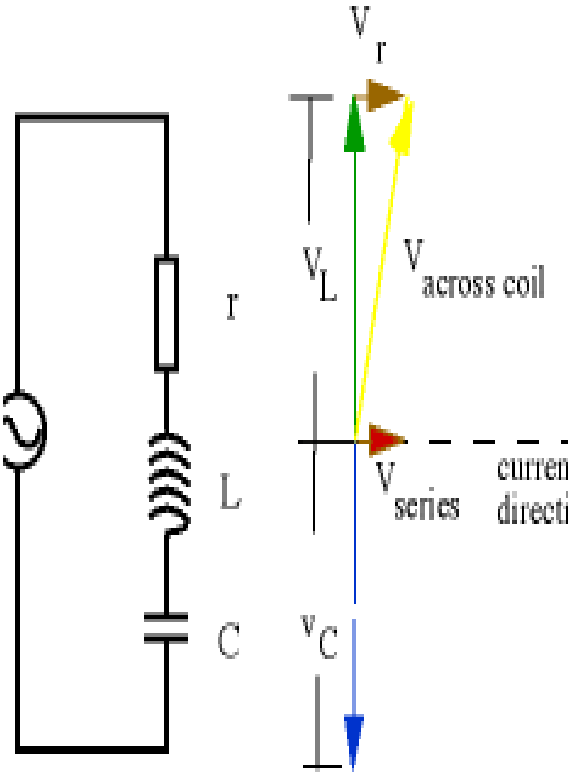
$\omega L = 1/\omega C$ so the frequency at which this occurs is

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$f_o = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

where ω_o and f_o are the angular and cyclic frequencies of resonance, respectively. At resonance, series impedance is a minimum, so the voltage for a given current is a minimum (or the current for a given voltage is a maximum).

This phenomenon gives the answer to our teaser question at the beginning. In an RLC series circuit in which the inductor has relatively low internal resistance r, it is possible to have a large voltage across the the inductor, an almost equally large voltage across capacitor but, as the two are nearly 180° degrees out of phase, their voltages almost cancel, giving a total series voltage that is quite small. This is one way to produce a large voltage oscillation with only a small voltage source. In the circuit diagram at right, the coil corresponds to both the inducance L and the resistance r. Why are they in series? Because the current flows through the coil and thus passes through both the inductance of the coil and its resistance.



You get a big voltage in the circuit for only a small voltage input from the power source. You are not, of course, getting something for nothing. The energy stored in the large oscillations is gradually supplied by the AC source when you turn on, and it is then exchanged between capacitor and inductor in each cycle. For more details about this phenomenon, and a discussion of the energies involved, go to LC oscillations.