

**Foundation of Technical Education**

**Al-Dour Technical Institute**

**Mechanical Department**

**2<sup>nd</sup> Stage**

**Training Package**  
**In**  
**Review of Strength of Materials**

**For**

**Students of second class**

**Mechanical Department/ Production**

**By**

**Nadum I. Naser**



# Overview

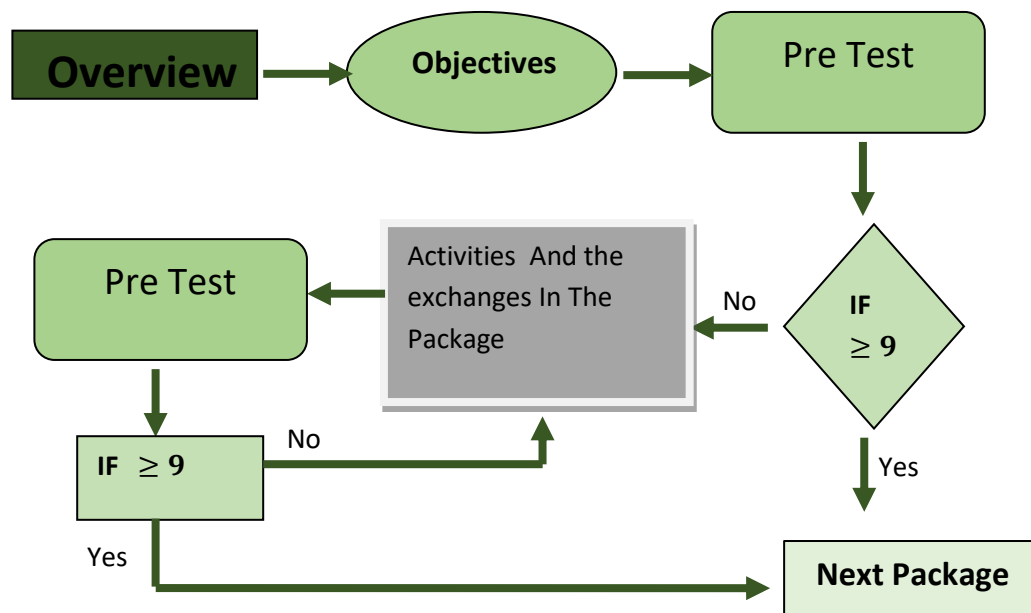
Strength of materials is a very important subjects to be studied in order to have a full knowledge about the characteristics of materials. For this eason I have designed this modular unit for this knowledge to be understood.

## Objectives :-

After studying the first modular unit , the student will be able to:-

- 1- Understand the strength of materials.
- 2- Define the stress.
- 3- Define the strain.
- 4- Distinguish between the elasticity and plasticity.

## Flow Chart:-



## Pre Test

Calculate the strain in a steel rod of square cross-section of side 2 cm when a force of 9800 N is applied and the young's modulus of elasticity is  $20.27 \times 10^9$  N/ m<sup>2</sup>?

## Notes

-10 degrees for the above question.

- Check your answers in key answer.

## The Text

### 1.stress :

The internal forces per unit area at any section of the body is known as stress.

$$S = \frac{F}{A} \quad (1)$$

Where :

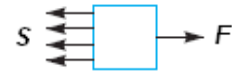
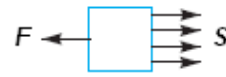
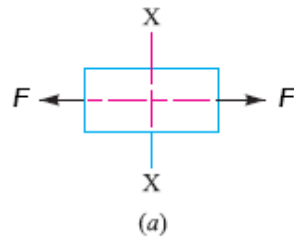
S – stress                      N/m<sup>2</sup>      or      Kg/cm<sup>2</sup>

F – Force                              N      or      Kg

A – Area                              m<sup>2</sup>      or      cm<sup>2</sup>

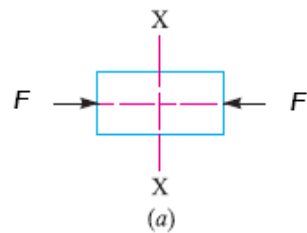
### Main types of stress

#### 1. Tensile stress



(b)

## 2. Compression stress



(b)

**Strain: the deformation per unit length**

$$\epsilon = \frac{\delta}{l}$$

where  $\delta$  – change in length

$\epsilon$  - strain

**Modulus of elasticity (E):**

Hook's law states that when a material is loaded within elastic limit, the stress is proportional to strain

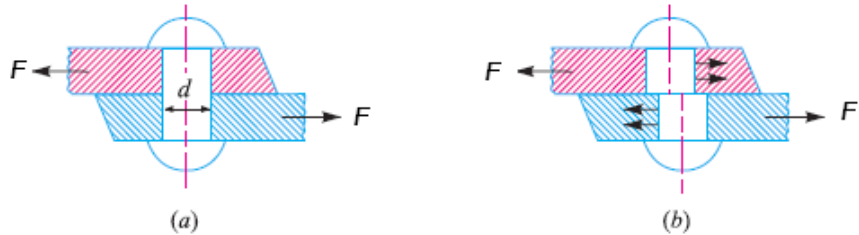
$$S \propto \epsilon$$

$$S = E \times \epsilon$$

Where  $E$  is young modulus of elasticity (constant)

$$E = \frac{S}{\epsilon} = \frac{F \cdot L}{A \cdot \delta} \quad \text{N/m}^2$$

### 3. Shear stress



$$\tau = \frac{F_s}{A_s}$$

The shear stress is directly proportional to shear strain

$$\tau \propto \phi$$

where

$\phi$  – shear strain

$\tau$  - shear stress

$$\tau = G \times \phi$$

$G$  – modulus of rigidity

$$\phi = \frac{\tau}{G} \quad (\text{N})$$

4. *Torsional shear stress*

$$\frac{\tau_s}{r} = \frac{T}{J}$$

$$J = \frac{\pi}{32} \times d^4$$

***For solid shafts***

$$J = \frac{\pi}{32} \times (D^4 - d^4)$$

***for hollow shafts***

$$\tau_s = \frac{16T}{\pi d^3}$$

**For solid shafts**

$$\tau_s = \frac{16T D}{\pi (D^4 - d^4)}$$

**for hollow shaft**

$\tau_s$  - max. shear stress

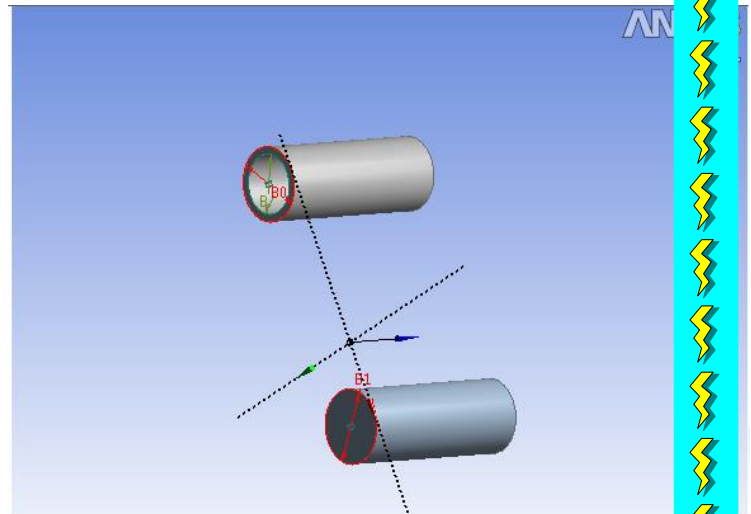
$T$  - torque or twisting moment

$r$  - Radius of the shaft

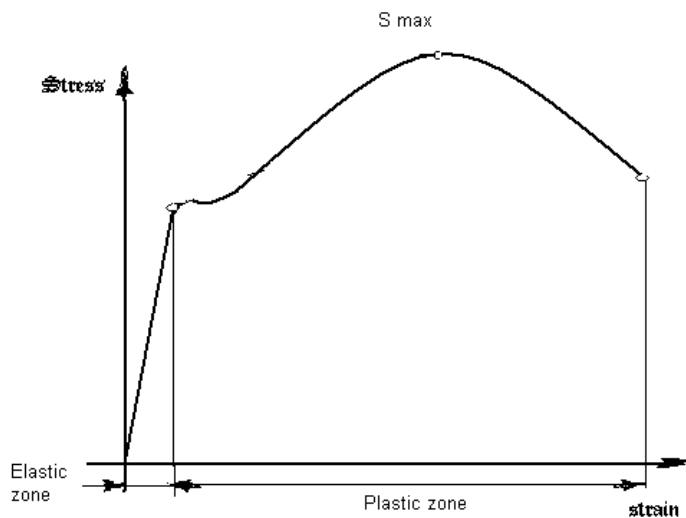
$d$  - Diameter of the shaft

$D$  - outer diameter of shaft

$J$  - polar moment of inertia



**Stress –Strain graph**



*Elastic zone: In this zone whenever the effected load lifted, the body recover to its original shape*

*Elasticity : it is the property of a material to regain its original shape after deformation when the external forces are removed*

*Plastic zone : in this zone whenever the effected load lifted, the body cannot recover to its original shape*

*Plasticity : it is the property of a material which retains the deformation produced under load permanantly..*

*Yield point : it is about the point which split the elastic and plastic zone.*



*Max. stress : Maximum stress which can the material sustain*

*Break stress : the stress which the failure of the material happen , always the maximum stress and the failure stress is the same.*

*Working stress : when designing machine parts its desirable to keep the stresses lower than the maximum stress at which failure of material takes place . this stress is known as working stress or design stress*

*Factor of safety :*

*It is defined as the ratio of the maximum stress to the working stress*

$$S.F = \frac{\text{max stress}}{\text{working stress}} = \frac{S_u}{S_w}$$

**Post Test**

An aluminium bar , length 10 meters , stressed by  $9.8 \times 10^6$  N/ m . if the Young's modulus of elasticity for aluminium  $6.752 \times 10^9$  N/ m<sup>2</sup>. Calculate the elongate of material.

## Key Answer

## Pre Test

$$\text{stress} = \frac{\text{force}}{\text{area}} = \frac{9800}{2 \times 10^{-2} \times 2 \times 10^{-2}} = 24.5 \times 10^6 \text{ N/m}^2$$

$$\text{strain} = \frac{\text{stress}}{\text{Young's modulus}} = \frac{24.5 \times 10^6}{20.27 \times 10^9} = 1.21 \times 10^{-3}$$

## Post Test

Stress  $\propto$  strain

Stress = Young's modulus  $\times$  strain

$$S = \epsilon \times E$$

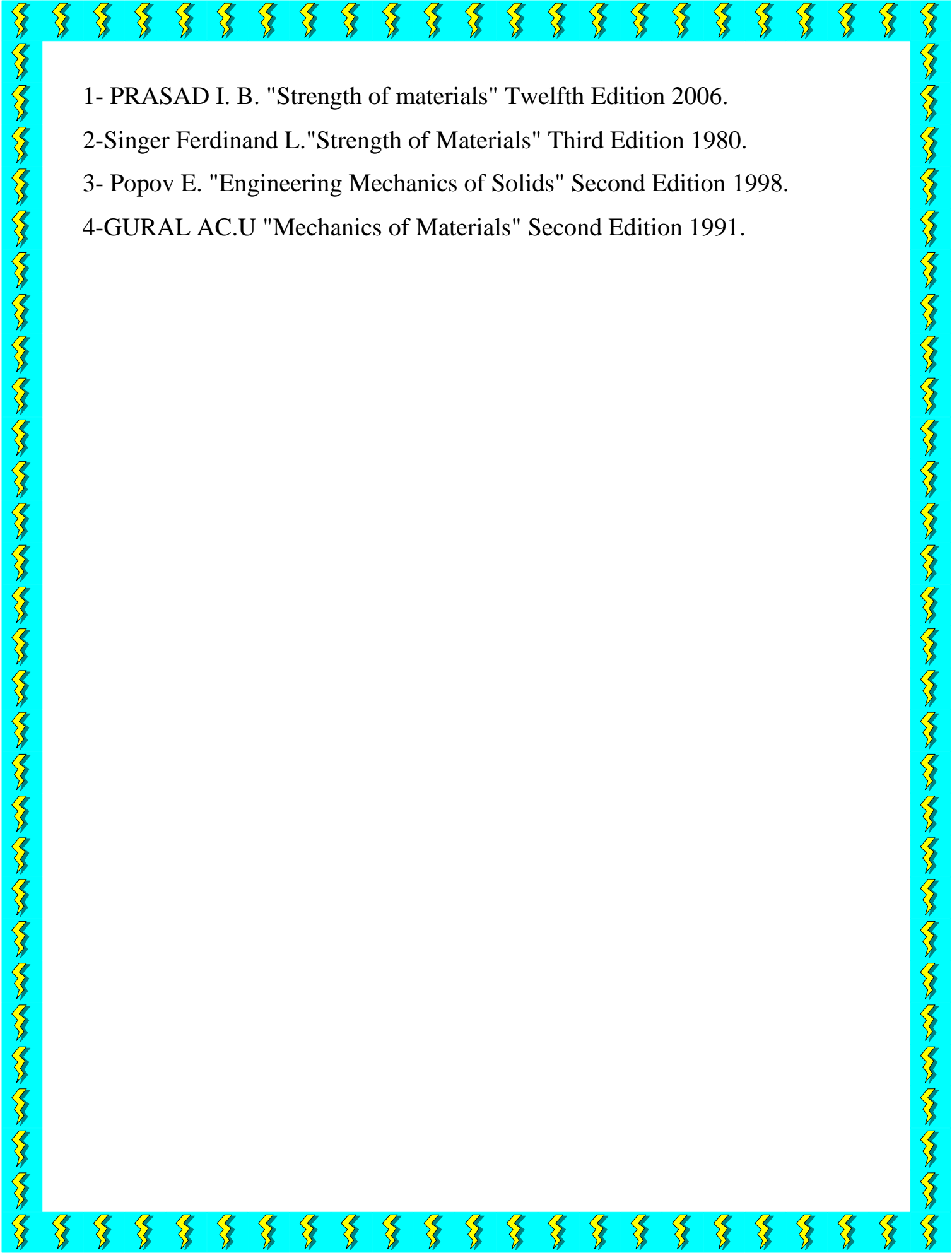
$$\text{strain } \epsilon = \frac{S}{E}$$

$$\text{strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\delta}{l_0}$$

$$\frac{\delta}{l_0} = \frac{S}{E}$$

$$\delta = \frac{S}{E} \times l_0 = \frac{9.8 \times 10^6 \times 10}{6.752 \times 10^9} = 14 \times 10^{-3} \text{ m}$$

## Reference

- 
- 1- PRASAD I. B. "Strength of materials" Twelfth Edition 2006.
  - 2-Singer Ferdinand L."Strength of Materials" Third Edition 1980.
  - 3- Popov E. "Engineering Mechanics of Solids" Second Edition 1998.
  - 4-GURAL AC.U "Mechanics of Materials" Second Edition 1991.

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**2<sup>nd</sup> Stage**

**Training Package**

**In**

# **Riveted Joints. Types of Riveted Joints, Design of Riveted Joints, Efficiency of Riveted Joints**

**For**  
**Students of second class**  
**Mechanical Department/ Production**  
**By**  
**Nadum I. Naser**



## **Overview**

A rivet is a short cylindrical bar with a head integral to it. The cylindrical portion of the rivet is called shank or body and lower portion of shank is known as tail.

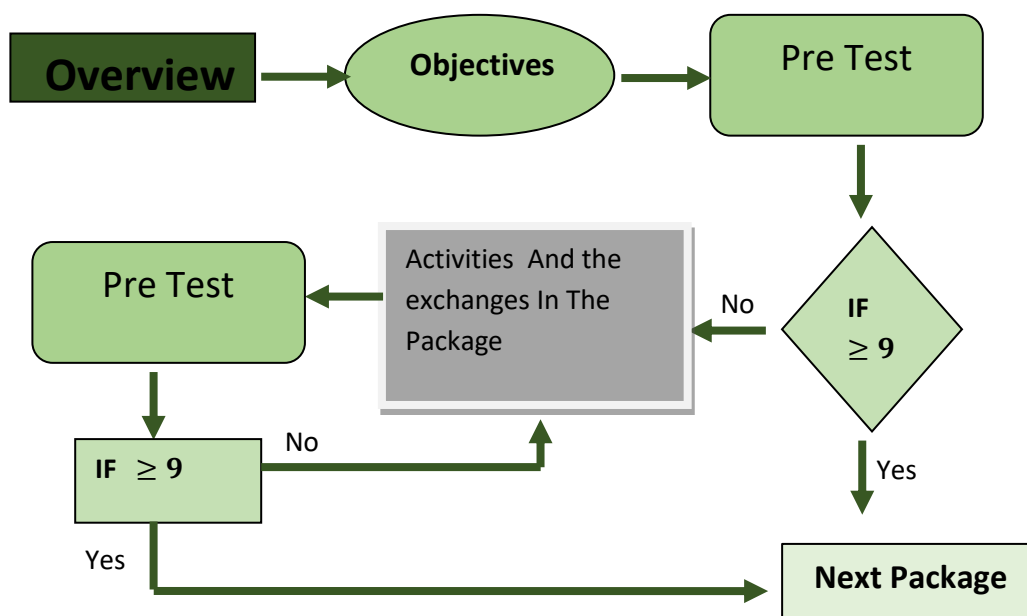
The rivets are used to make permanent fastening between the plates such as in structural work, ship building, bridges, tanks and boiler shells. The rivet joints are widely used for joining light metals.

## Objectives :-

After studying the first modular unit , the student will be able to:-

- 1-Recognize the types of rivet joints.
- 2- Calculate the stress on the rivet.

## Flow Chart:-



## Pre Test

Find the efficiency of the following riveted joints:

- 1.Single riveted lap joint of 6 mm plates with 20 mm diameter rivets having a pitch of 50 mm.
- 2.double riveted lap joint of 6 mm plates with 20 mm diameter rivets having a pitch of 65 mm.

**Assume:**

**Permissible tensile stress in plate = 120 MPa**

**Permissible shearing stress in rivets = 90 MPa**

**Permissible crushing stress in rivets = 180 MPa**

### **Notes**

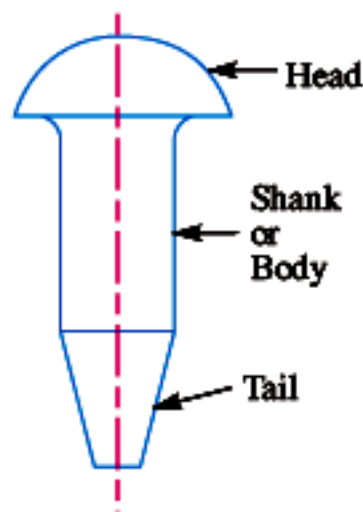
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### **The Text**

A rivet is a short cylindrical bar with a head integral to it. The cylindrical portion of the rivet is called shank or body and lower portion of shank is known as tail.

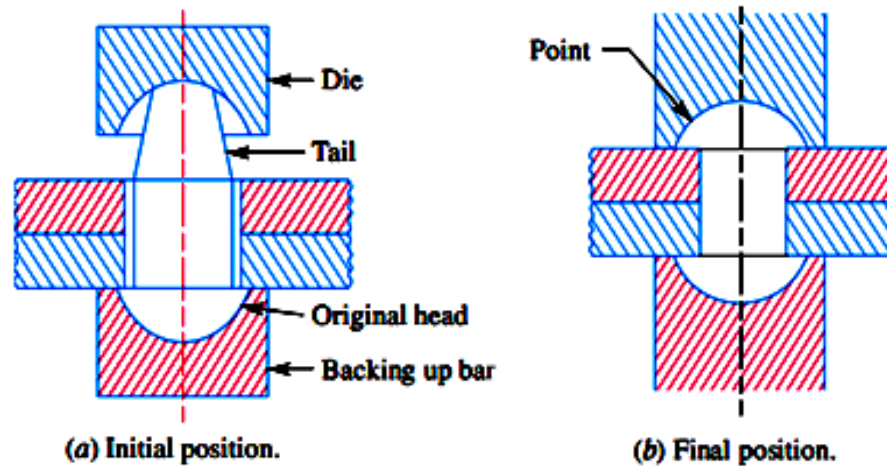
The rivets are used to make permanent fastening between the plates such as in structural work, ship building, bridges, tanks and boiler shells. The rivet joints are widely used for joining light metals.



**Method of riveting**

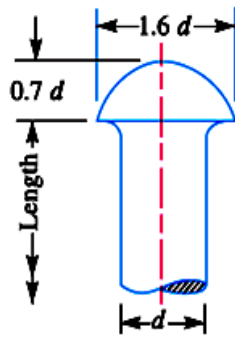
1-drill the plate that would be riveted , the diameter of hole is usually diameter of rivet

2-the red hot rivet is introduced into the plates and the second head is then formed, this may be done by hand or by a riveting machine.

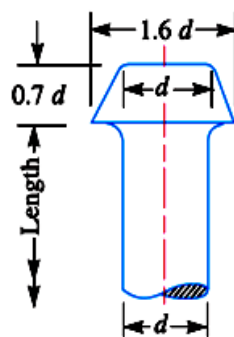


Types of riveted heads

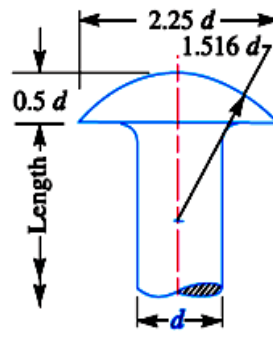




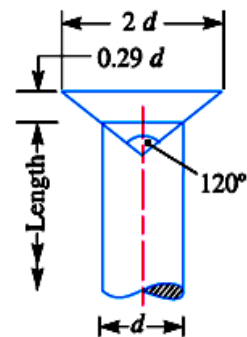
(a) Snap head.



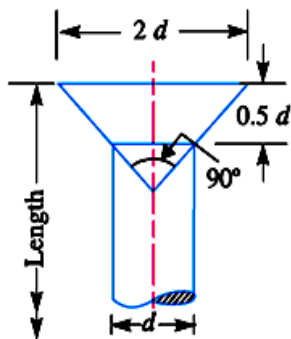
(b) Pan head.



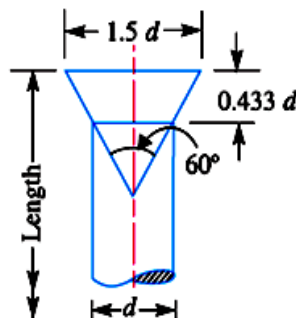
(c) Mushroom head.



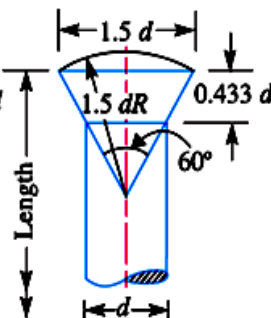
(d) Counter sunk head 120°.



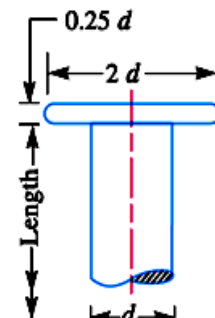
(e) Flat counter sunk head 90°.



(f) Flat counter sunk head 60°.



(g) Round counter sunk head 60°.

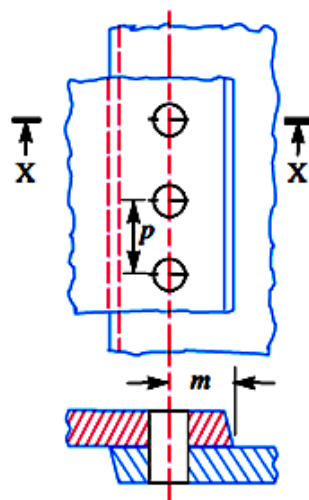


(h) Flat head.

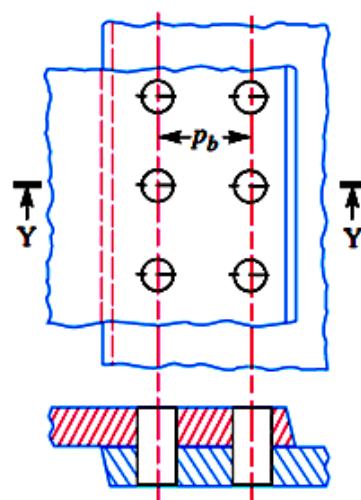
## Types of riveted joints

### 1- Lap joint

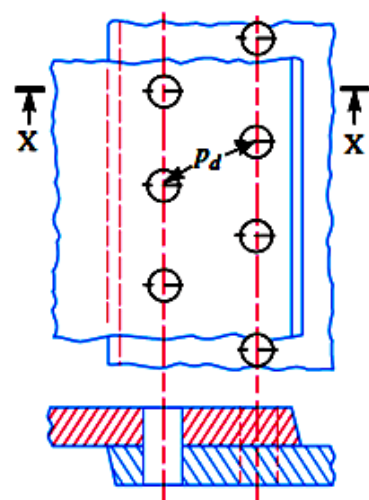
A lap joint is that in which one plate overlaps the other and the two plates are then riveted together



(a) Single riveted lap joint.



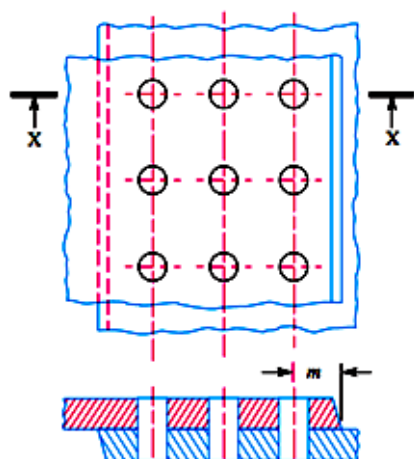
(b) Double riveted lap joint (Chain riveting).



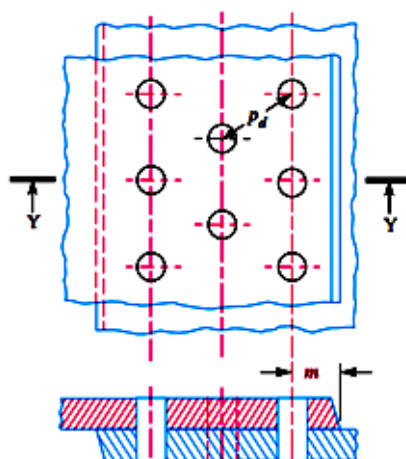
(c) Double riveted lap joint (Zig-zag riveting).

### 2-Butt joint

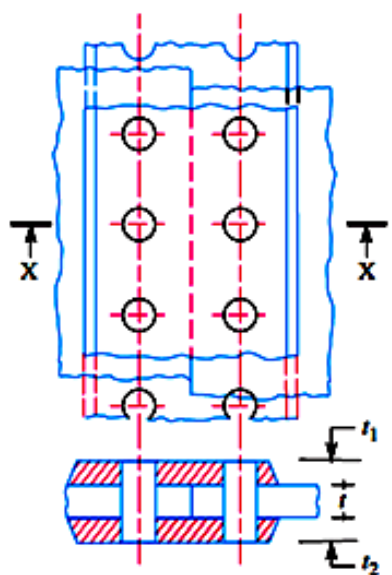
A butt joint is that in which the main plates are kept in alignment butting (touching) each other on one side or on both sides of the main plates.

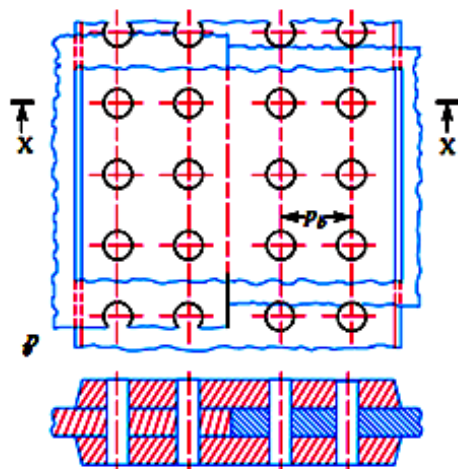


(a) Chain riveting.

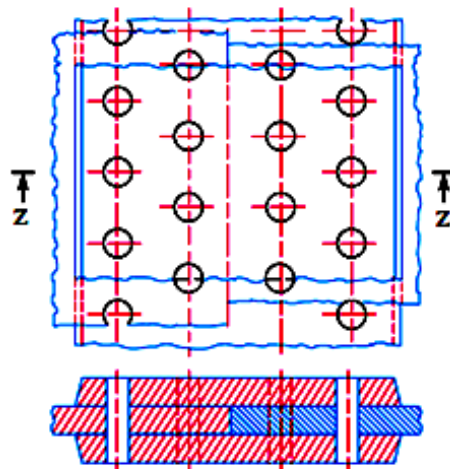


(b) Zig-zag riveting.

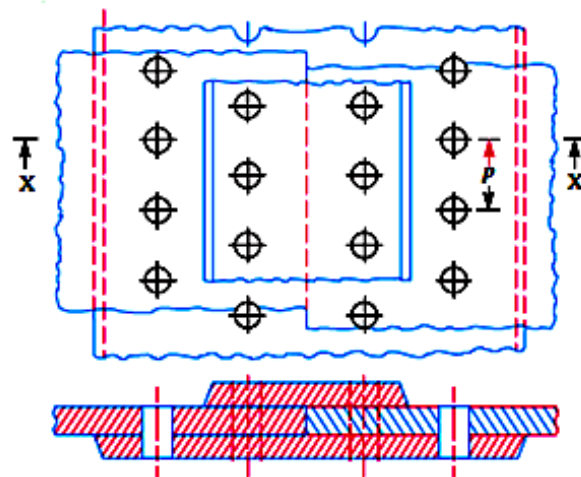




(a) Chain riveting.



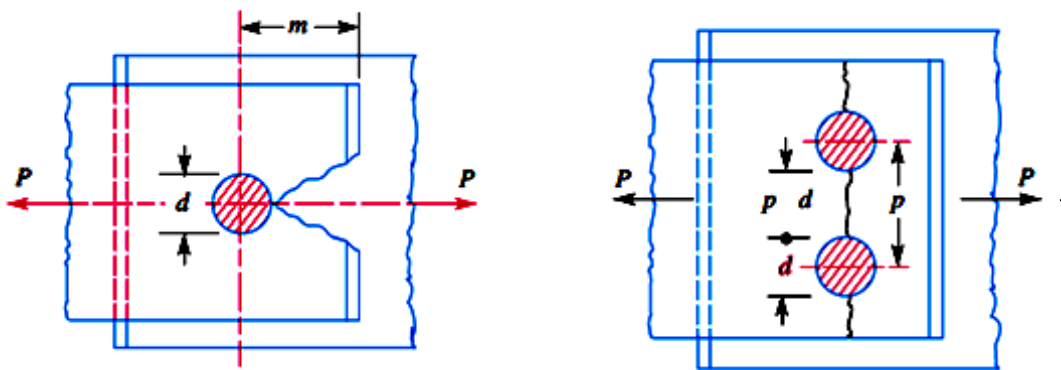
(b) Zig-zag riveting.



### Technical terms

- 1- Pitch( $p$ ): it is the distance from the center of one rivet to the center of the next, measured parallel to the center.
- 2- Bach pitch( $P_b$ ):it is the perpendicular distance between the center lines of the successive rows.
- 3- Diagonal pitch( $P_d$ ):it is the distance between the center of rivet in adjacent rows of zig - zag riveted joints.
- 4- Margin( $M$ ):it is the distance between the center of rivet hole and the nearest edge of the plate.

## Failures of riveted joints



### 1-Tearing of the plate at the edge:

This can be avoided by keeping the margin A ( $m = 1.5 d$ )

### 2-Tearing of the plate across a row of rivets:

$$A_t = (P - d) \times t$$

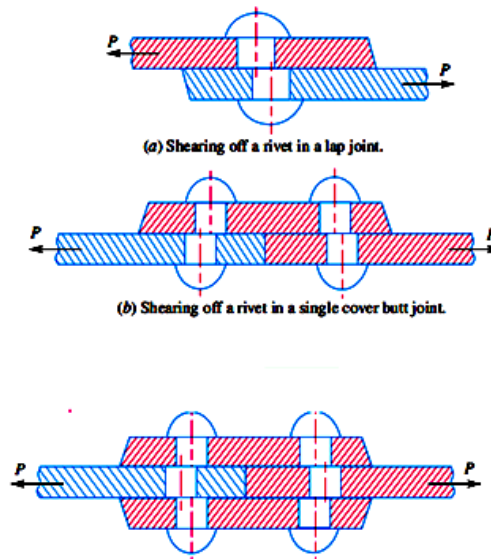
$$F_t = A_t \times S_t = (P - d) \times t \times S_t$$

$t$  - thickness of the plate

$d$  - diameter of hole

$S_t$  - Tensile stress for the plate material

### 3- Shearing of the rivets:



$$= \frac{\text{Tangential force}}{\text{Resisting area}} \tau$$

$$A = \frac{\pi}{4} \times d^2$$

$$\tau = \frac{F}{A} = \frac{F}{\frac{\pi}{4} \times d^2} = \frac{4F}{\pi d^2}$$

$\tau$  - shear stress on the rivet cross section

d- diameter of the rivet

when the shearing takes place at two cross- sections of the rivet, then the rivets are said to be in (double shear )

$$A = 2 \times \frac{\pi}{4} \times d^2$$

sections

for double rivet cross-

$$\tau = \frac{F}{A} = \frac{F}{2 \times \frac{\pi}{4} \times d^2} = \frac{2F}{\pi d^2}$$

$$F_s = \frac{\pi}{4} \times d^2 \times n \times \tau$$

shear)

Shearing force on the rivet(single

$$F_s = \frac{\pi}{2} \times d^2 \times n \times \tau$$

rivet(double shear)

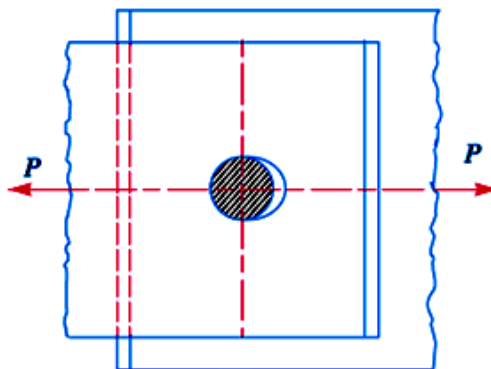
shearing force on the double

n- number of rivets per pitch length

note:

All lap joints and single cover butt joints are in single shear , while the butt joints with double cover plates are in double shear.

#### 4- Crushing of the rivet



$$A_c = d \times t$$

$$F_c = d \times t \times n \times S_c$$

**$d$  - diameter of the rivet**

**$t$  - thickness of the plate**

**$n$  - number of rivets per pitch length**

**$S_c$  - crushing stress**

**5-Efficiency of the riveted joints:**

**The efficiency of the riveted joints is defined as a ratio of strength of riveted joints to the strength of unriveted or (solid) plate.**

$$\eta = \frac{\text{least of } F_t, F_s \text{ and } F_c}{\text{strength of unriveted plate}}$$

$$\eta = \frac{\text{least of } F_t, F_s \text{ and } F_c}{P \times t \times S_t}$$

**$F_t$  -Tearing resistance of plate**

**$F_s$  -Shearing resistance of rivet**

**$F_c$  -crushing resistance of rivet**

**$P$  – Pitch**

**$t$  – thickness of the plate**

**$S_t$  -Tensile strength(stress)of the plate**



## Post Test

A double riveted double cover butt joint in plate 20 mm thick is made with 25 mm diameter rivets at 100 mm pitch .the permissible stresses are :

$$S_t=120 \text{ N/mm}^2 \quad \tau = 100 \text{ N /mm}^2 \quad S_c=150 \text{ N / mm}^2$$

Find the efficiency of joint , taking the strength of rivet in double shear as twice than that of single shear.

## Key Answer

### Pre Test

1.Efficiency of the first joint:

$$P=50 \text{ mm} \quad t = 6 \text{ mm} \quad d = 20 \text{ mm} \quad S_t = 120 \text{ MPa} = 120 \text{ N / mm}^2$$

$$\text{N / mm}^2 \quad S_c = 180 \text{ MPa} = 180 \text{ N / mm}^2 \quad \tau = 90 \text{ MPa} = 90$$

(i)tearing resistance of the plate

$$F_t = (P - d) \times t \times S_t = (50 - 20) \times 6 \times 120 = 21600 \text{ N}$$

(ii) shearing resistance of the rivet:

$$F_s = \frac{\pi}{4} \times d^2 \times n \times \tau$$

$$F_s = \frac{\pi}{4} \times 20^2 \times 90 = 28287 \text{ N}$$

(iii)crushing resistance of the rivet:

$$F_c = d \times t \times n \times S_c$$

$$F_c = 20 \times 6 \times 180 = 21600 \text{ N}$$

$$\eta = \frac{\text{least of } Ft, Fs \text{ and } Fc}{P \times t \times S_t}$$

$$P \times t \times S_t = 50 \times 6 \times 120 = 36000 \text{ N}$$

Least resistance is = 21600 N

$$\eta = \frac{21600}{36000} = 0.60 \quad \text{or} \quad 60\%$$

2. Efficiency of the second joint:

$$P = 65 \text{ mm} \quad t = 6 \text{ mm} \quad d = 20 \text{ mm} \quad S_t = 120 \text{ MPa} = 120 \text{ N / mm}^2$$

$$\text{N / mm}^2 \quad S_c = 180 \text{ MPa} = 180 \text{ N / mm}^2 \quad \tau = 90 \text{ MPa} = 90$$

(i) tearing resistance of the plate

$$F_t = (P - d) \times t \times S_t = (65 - 20) \times 6 \times 120 = 32400 \text{ N}$$

(ii) shearing resistance of the rivet:

$$F_s = \frac{\pi}{4} \times d^2 \times n \times \tau$$

$$F_s = \frac{\pi}{4} \times 20^2 \times 2 \times 90 = 56548 \text{ N}$$

(iii) crushing resistance of the rivet:

$$F_c = d \times t \times n \times S_c$$

$$F_c = 20 \times 6 \times 2 \times 180 = 43200 \text{ N}$$

$$\eta = \frac{\text{least of } Ft, Fs \text{ and } Fc}{P \times t \times S_t}$$

Least resistance is = 32400 N

$$P \times t \times S_t = 65 \times 6 \times 120 = 46800 \text{ N}$$

$$\eta = \frac{32400}{46800} = 0.692 \text{ or } 69.2 \%$$

## Post Test

$$F_t = (P - d) \times t \times S_t = (100 - 25) \times 20 \times 120 = 180000N$$

$$F_s = 2 \times \frac{\pi}{4} \times d^2 \times n \times \tau = 2 \times \frac{\pi}{4} \times 25^2 \times 2 \times 100 = 196349N$$

$$F_c = d \times t \times n \times S_c = 25 \times 20 \times 2 \times 150 = 150000N$$

*∴ least of these three values  $F_c = 150000N$*

$$\eta = \frac{150000}{P \times t \times S_t} = \frac{150000}{100 \times 20 \times 120} = 0.625 \text{ or } 62.5 \%$$

## Reference

R. S. Khurmi, J. K. Gupta, "Theory of machine"

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2<sup>nd</sup> Stage

**Training Package**  
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**Welded Joints Types of welding Joints ,**  
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**Overview**

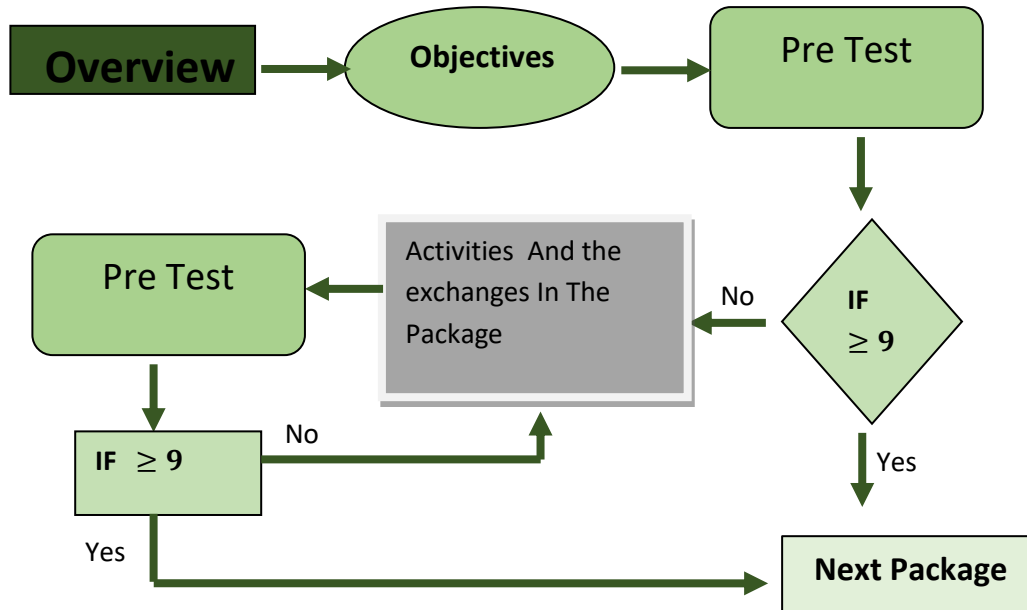
A welded joint is a permanent joint which is obtained by the fusion of the edges of the two parts to be joined together, with or without the application of pressure and a filler material. The heat required for the fusion of the material may be obtained by burning of gas (in case of gas welding) or by an electric arc (in case of electric arc welding) . the later method is extensively used because of greater speed of welding.

### **Objectives :-**

After studying the first modular unit , the student will be able to:-

- 1-Define Types of welding joints.
- 2- Calculate the length of weld.

### **Flow Chart:-**



### **Pre Test**

A plate 100 mm wide and 10 mm thick is to be welded to another plate by means of double parallel fillets. The plates are subjected to a static

load of 80 kN. Find the length of weld if the permissible shear stress in the weld does not exceed 55 MPa.

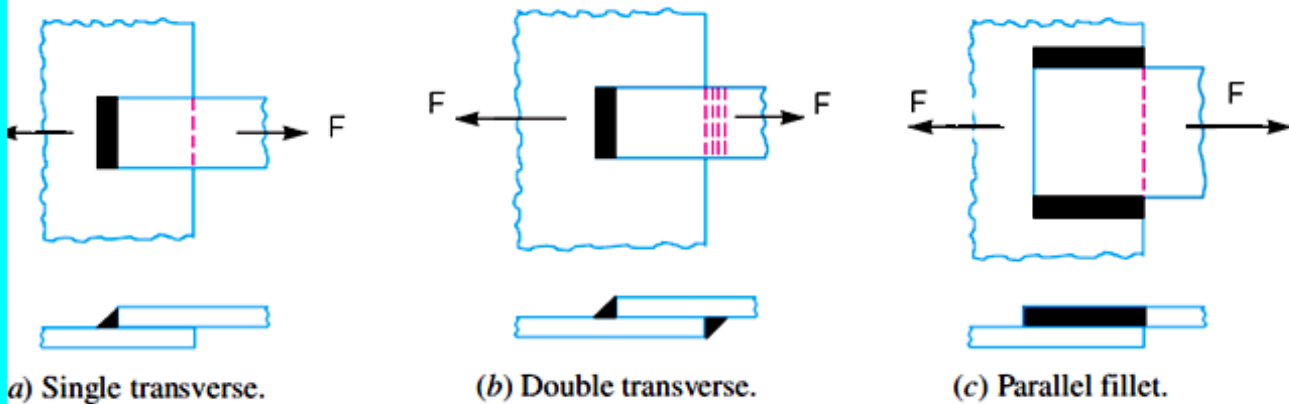
### Notes

-10 degrees for the above question.

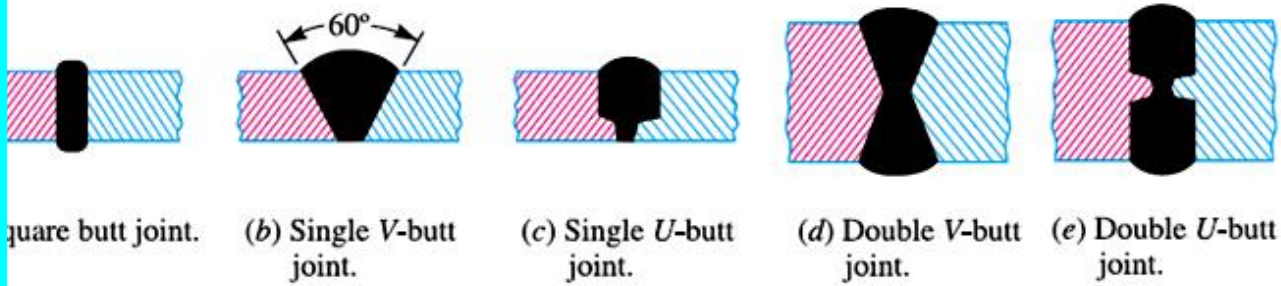
- Check your answers in key answer.

### The Text

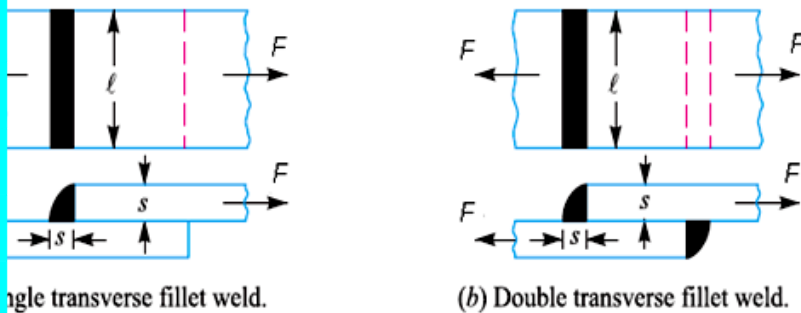
1. lap joint or fillet joint



2. Butt joint



### *Strength of transverse fillet welded joints*

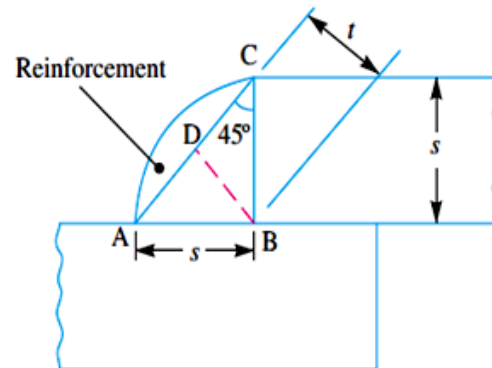


$t$  = Throat thickness ( $BD$ ),  
 $s$  = Leg or size of weld,  
 $t$  = Thickness of plate, and  
 $l$  = Length of weld,

$$t = s \times \sin 45^\circ = 0.707 s$$

$\therefore$  Minimum area of the weld or throat area,

$$A = \text{Throat thickness} \times \text{Length of weld} = t \times l = 0.707 s \times l$$



Tensile force of the joint for single fillet

$$F_t = A \times S_t$$

$$F_t = 0.707 s \times l \times S_t$$

Where

$S_t$  - allowable tensile stress for weld metal

Tensile force of the joint for double fillet

$$F_t = 2 \times 0.707 s \times l \times S_t$$

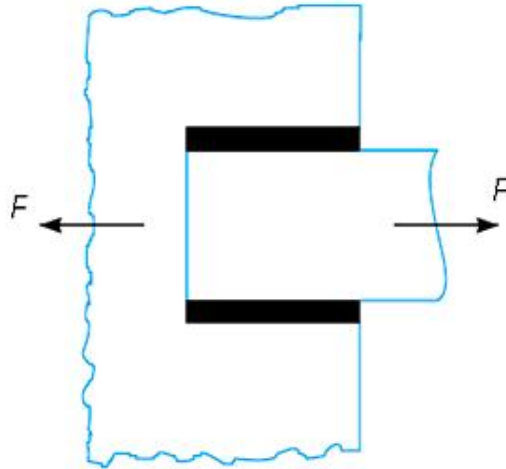
$$F_t = 1.414 s \times l \times S_t$$

***Strength of parallel fillet welded joints***



$$F = \text{Throat area} \times \text{Allowable shear stress} = 0.707 s \times l \times \tau$$

$$F = 2 \times 0.707 \times s \times l \times \tau = 1.414 s \times l \times \tau$$

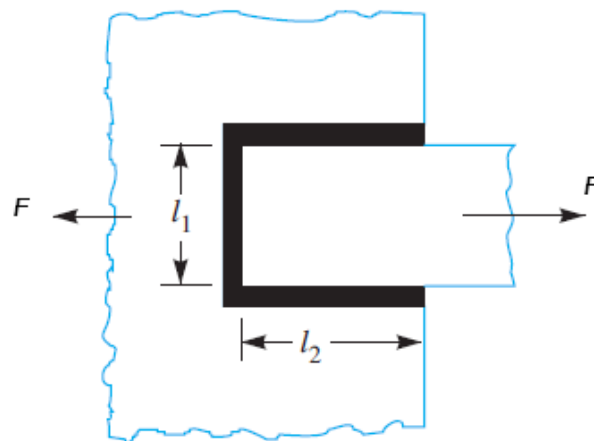


(a) Double parallel fillet weld.

1. If there is a combination of single transverse and double parallel fillet welds. Then the strength of the joint is given by the sum of strengths of single transverse and double parallel fillet welds.

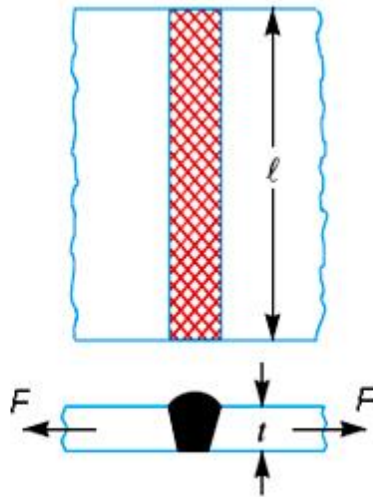
Mathematically, 
$$F = 0.707s \times l_1 \times \tau + 1.414 s \times l_2 \times \tau$$

in order to allow for starting and stopping of the bead, 12.5 mm should be added to the length of each weld.

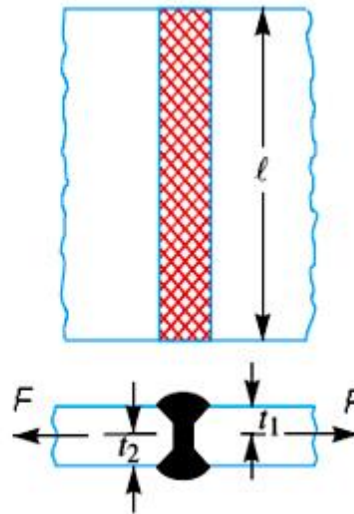


(b) Combination of transverse and parallel fillet weld.

## *Strength of butt joints*



(a) Single V-butt joint.



(b) Double V-butt joint.

In case of butt joint, the length of leg or size of weld is equal to the throat thickness which is equal to thickness of plates.

Tensile strength of the butt joint (single-V or square butt joint),

$$F = t \times l \times St \quad l = \text{Length of weld. It is generally equal to the width of plate.}$$

Tensile strength for double-V or square butt joint

$$F = (t_1 + t_2) l \times St \quad t_1 = \text{Throat thickness at the top, and}$$

$$t_2 = \text{Throat thickness at the bottom.}$$

## *Stress concentration factor*















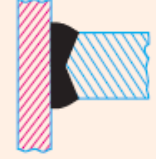

When the parts of welded joint are subjected to fatigue loading (dynamic loading) the S.C.F as given below should be taken into account.



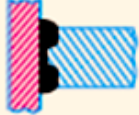

























Type of joint	Stress concentration factor
1. Reinforced butt welds	1.2
2. Toe of transverse fillet welds	1.5
3. End of parallel fillet weld	2.7
4. T-butt joint with sharp corner	2.0

$$S_t = \frac{S_t}{S.C.F} \quad , \quad \tau = \frac{\tau}{S.C.F}$$

Note: for static loading and any type of joint , S.C.F = 1

### **Basic weld symbols**

S. No.	Form of weld	Sectional representation	Symbol
1.	Fillet		
2.	Square butt		
3.	Single-V butt		
4.	Double-V butt		
5.	Single-U butt		
6.	Double-U butt		
7.	Single bevel butt		
8.	Double bevel butt		

S. No.	Form of weld	Sectional representation	Symbol
9.	Single-V butt		
10.	Double-V butt		
11.	Bead (edge or seal)		
12.	Stud		
13.	Sealing run		
14.	Spot		
15.	Seam		
16.	Mashed seam	 Before      After	
17.	Plug		
18.	Backing strip		
19.	Stitch		
20.	Projection	 Before      After	
21.	Flash	 Rod or bar      Tube	
22.	Butt resistance or pressure (upset)	 Rod or bar      Tube	

## **Post Test**

A plate 100 mm wide and 12.5 mm thick is to be welded to another plate by means of parallel fillet welds. The plates are subjected to a load of 50 KN. Find the length of weld so that the maximum stress does not exceed 56 MPa. Consider the joint first under static loading and then under fatigue loading.

## **Key Answer**

### **Pre Test**

Let  $l$  – length of weld

s- size of weld = plate thickness = 10 mm

$$F = 1.414 \times s \times l \times \tau$$

$$80 \times 10^3 = 1.414 \times 10 \times l \times 55 = 788 l$$

$$l = \frac{80 \times 10^3}{788} = 103 \text{ mm}$$

*Adding 12.5 mm for starting and stopping of the weld run, we have*

$$l = 103 + 12.5 = 115.5 \text{ mm} .$$

## **Post Test**

1.length of weld under static loading

Let  $l$  – length of weld

s- size of weld = plate thickness = 12.5 mm

$$F = 1.414 \times s \times l \times \tau$$

$$50 \times 10^3 = 1.414 \times 12.5 \times l \times 56 = 990 \ l$$

$$l = \frac{50 \times 10^3}{990} = 50.5 \text{ mm}$$

*adding 12.5 mm for starting and stopping of the weld run, we have*

$$l = 50.5 + 12.5 = 63 \text{ mm}$$

2.length of weld for fatigue loading

From table we find that the stress concentration factor is for parallel fillet welding is 2.7

$$\therefore \text{permissible shear stress} \quad \tau = \frac{56}{2.7} = 20.74 \text{ N/mm}^2$$

$$F = 1.414 \times s \times l \times \tau$$

$$50 \times 10^3 = 1.414 \times 12.5 \times l \times 20.74 = 367 \ l$$

$$l = \frac{50 \times 10^3}{367} = 136.2 \text{ mm}$$

*adding 12.5 mm for starting and stopping of the weld run, we have*

$$l = 136.2 + 12.5 = 148.7 \text{ mm.}$$

## **Reference**

R. S. Khurmi, J. K. Gupta, "Theory of machine"



**Foundation of Technical Education**

**Al-Dour Technical Institute**

**Mechanical Department**

**2<sup>nd</sup> Stage**

**Training Package**

**In**

**Screwed Joints, Design of Bolts for Fastening ,  
Design of Bolts for Power Transition .**

**For**

**Students of second class**

**Mechanical Department/ Production**

**By**

**Nadum I. Naser**



# Overview

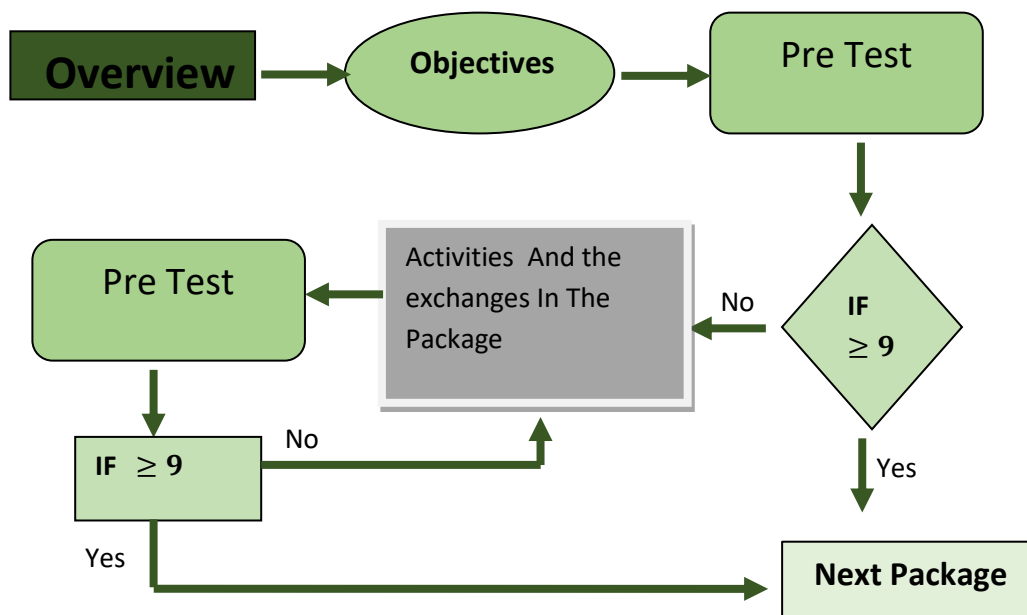
Power screws are used to convert rotary motion into translatory motion. In most of power screws the nut has axial motion against the resisting axial force while the screw rotates in its bearing. In some screws, the screw rotates and moves axially against the resisting force while the nut is stationary, and in others the nut rotates while the screw moves axially with no rotation.

## Objectives :-

After studying the first modular unit , the student will be able to:-

- 1- Define Pitch and lead in the screw.
- 2- calculate tensile stress in the screw.
- 3- calculate torsion stress in the screw.
- 4- calculate shear stress in the screw.
- 5- calculate crush stress in the screw.

## Flow Chart:-





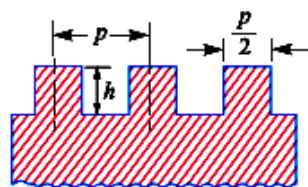
## Pre Test

A vertical screw with single start square threads of 50 mm mean diameter and 12.5 mm pitch is raised against a load of 10 kN by means of a hand wheel, the boss of which is threaded to act as a nut. The axial load is taken up by a thrust collar which supports the wheel boss and has a radius of 30 mm. The coefficient of friction is 0.15 for the screw and 0.18 for the collar. If the tangential force applied by each hand to the wheel is 100 N, find the diameter of the hand wheel.

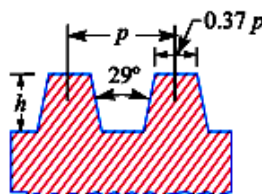
## Notes

- 10 degrees for the above question.
- Check your answers in key answer.

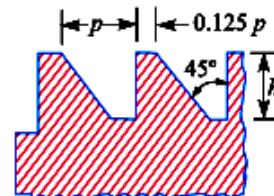
## The Text



$h = 0.5 p$   
(a) Square thread.



$h = 0.5 p + 0.25 \text{ mm}$   
(b) Acme thread.



$h = 0.75 p$   
(c) Buttress thread.

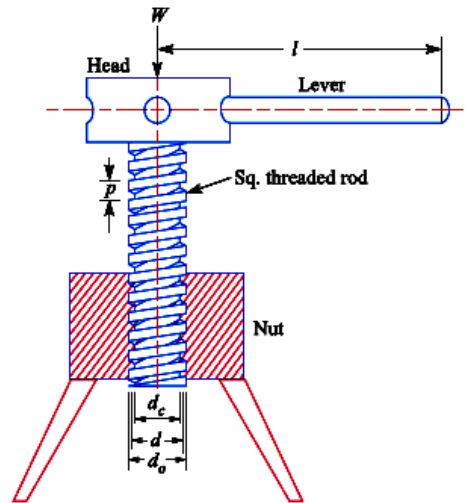
1. square thread

2. acme thread

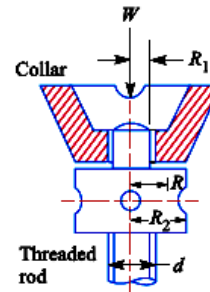
3. buttress  
thread

Torque required to raise load by square threaded screws:

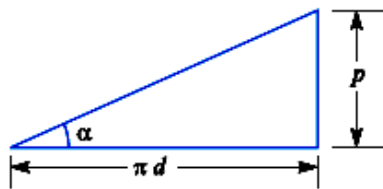
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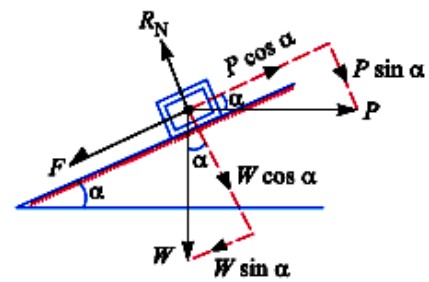
Screw jack.



Thrust collar.



Development of a screw.



Forces acting on the screw.

$p$  – pitch of the screw

$d$  – mean diameter of the screw  $d = \frac{d_o + d_c}{2}$

$\alpha$  – helix angle

$$\tan \alpha = \frac{p}{\pi d}$$

$W$  – load to be lifted

$P$  – force applied at the circumference of the screw

$\mu$  – coefficient of friction

$$\mu = \tan \phi$$

$\phi$  – friction angle

$$F = \mu \times R_N$$

$$P = W \tan (\alpha + \phi)$$

Torque required to overcome friction between screw and nut

$$T_1 = P \times \frac{d}{2} = W \tan (\alpha + \phi) \times \frac{d}{2}$$

When the axial load is taken up by a thrust collar, so that the load does not rotate with the screw, then the torque required to overcome the friction at the collar:

$$T_2 = \mu_1 W R$$

$$R = \frac{R_1 + R_2}{2}$$

$\mu_1$  – friction of the collar

$R_1, R_2$  – inside and outside radii of the collar

Total torque required to overcome friction :

$$T = T_1 + T_2$$

If the effort  $P_1$  applied at the end of a lever of arm length  $l$ , then the total torque required to overcome friction must be equal to the torque applied at the end of lever.

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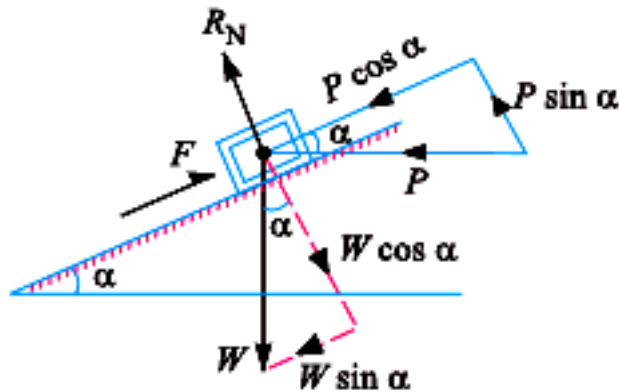
$$T = P \times \frac{d}{2} = P_1 \times l$$

$$P_1 = \frac{P \times d}{2l}$$

The mechanical advantage is the ratio of the load lifted ( $W$ ) to the effort applied ( $P_1$ ) at the end of the lever

$$\begin{aligned} M.A &= \frac{W}{P_1} = \frac{W \times 2l}{P \times d} \\ &= \frac{W \times 2l}{W \tan(\alpha + \phi)} = \frac{2l}{\tan(\alpha + \phi)} \end{aligned}$$

Torque required to lower load by square threaded screws:



$$P = W \tan(\phi - \alpha)$$

Torque required to overcome friction between the screw and nut

$$T_1 = P \times \frac{d}{2} = W \tan(\phi - \alpha) \times \frac{d}{2}$$

Note1 : when  $\alpha > \phi$

$$\text{Then } P = W \tan(\alpha - \phi)$$

Note2: the mean diameter  $d$  of the screw is  $d = \frac{d_o + d_c}{2} = d_o - \frac{p}{2} = d_c + \frac{p}{2}$

$d_o$  – nominal diameter

$d_c$  – core diameter

## Post Test

An electric motor driven power screw moves a nut in a horizontal plane against a force of 75 kN at a speed of 300 mm/min. The screw has a single square thread of 6 mm pitch, on a major diameter of 40 mm. The coefficient of friction at screw threads is 0.1. Find the power of the motor.

## Key Answer

## Pre Test

$$\tan \alpha = \frac{p}{\pi d} = \frac{12.5}{\pi \times 50} = 0.08$$

$$\mu = \tan \phi = 0.15$$

$$P = W \tan(\alpha + \phi) = W \left( \frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi} \right)$$
$$= 10 \times 10^3 \left( \frac{0.08 + 0.15}{1 - 0.08 \times 0.15} \right) = 2328 \text{ N}$$

$$T = P \times \frac{d}{2} + \mu_1 W R = 2328 \times \frac{50}{2} + 0.18 \times 10 \times 10^3 \times 30$$
$$= 58200 + 54000 = 112200 \text{ N.mm}$$

let  $D_1$  – diameter of hand wheel

$\therefore$  torque on hand wheel

$$T = 2 p_1 \times \frac{D_1}{2} = 2 \times 100 \times \frac{D_1}{2} = 100 D_1 \text{ N.mm}$$

$$112200 = 100 D_1$$

$$D_1 = \frac{112200}{100} = 1122 \text{ mm} = 1.122 \text{ m}$$

## Post Test

$$d = d_o - \frac{p}{2} = 40 - \frac{6}{2} = 37 \text{ mm}$$

$$\tan \alpha = \frac{p}{\pi d} = \frac{6}{\pi \times 37} = 0.0516$$

$$\mu = \tan \phi = 0.1$$

$$P = W \tan(\alpha + \phi) = W \left( \frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi} \right)$$

$$= 75 \times 10^3 \left( \frac{0.0516 + 0.1}{1 - 0.0516 \times 0.1} \right) = 11.43 \times 10^3 \text{ N}$$

$$T = P \times \frac{d}{2} = 11.43 \times 10^3 \times \frac{37}{2} = 211.45 \times 10^3 \text{ N.mm} = 211.45 \text{ N.m}$$

$$N = \frac{\text{Speed in mm/min}}{\text{Pitch in mm}} = \frac{300}{6} = 50 \text{ R.P.M}$$

$$\text{Angular speed } w = \frac{2\pi N}{60} = \frac{2\pi \times 50}{60} = 5.24 \text{ rad/s}$$

$$\text{Power of the motor } P = T \times w = 211.45 \times 5.24 = 1108 \text{ W} = 1.108 \text{ KW}$$

## Reference

R. S. Khurmi, J. K. Gupta, "Theory of machine"

**Foundation of Technical Education**

**Al-Dour Technical Institute**

**Mechanical Department**

**2<sup>nd</sup> Stage**

**Training Package**

**In**

**Keyed Joints , Types of Key , Design of Sunk Key**

**For**

**Students of second class**

**Mechanical Department/ Production**

**By**

**Nadum I. Naser**



## **Overview**

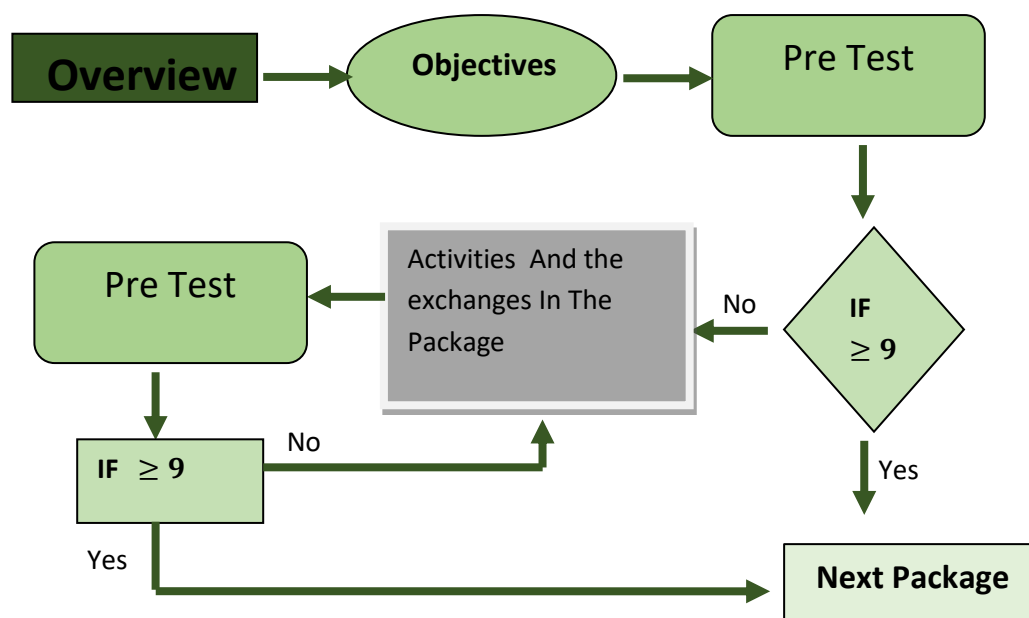
key : it is a piece of metal has different forms used to prevent relative motion between a shaft and the connected member (hub) through which torque is being transmitted

## Objectives :-

After studying the first modular unit , the student will be able to:-

- 1-Define Types of key.
- 2-Calculate Stresses in keys ∩

## Flow Chart:-



## Pre Test

A sunk key of 14 mm width . 20 mm thick and 50 mm length ,is required to transmit torque of 14000 kg.cm from a solid shaft of 4 cm diameter .Calculate if the length of key is enough or not, when the design stresses of the key are for shear  $560 \text{ kg/cm}^2$  , and crushing stress  $1680 \text{ kg/cm}^2$  . Also calculate the enough length when the original length of the key cannot sustain the above stress

## Notes



-10 degrees for the above question.

- Check your answers in key answer.

### **The Text**

1.sunk keys

2.saddle keys

3.tangent keys

4.round keys

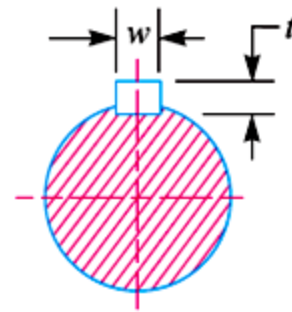
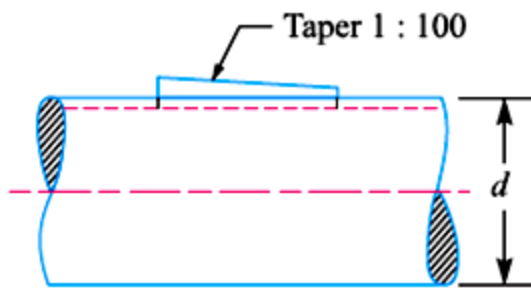
5.splines

### **Sunk keys**

The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub.

The sunk keys are the following types:

**a. Rectangular sunk keys:**



$$w = \frac{d}{4}$$

$$t = \frac{2w}{3} = \frac{2 \times \frac{d}{4}}{3} = \frac{d}{6}$$

w – width of key

t – thickness of key

d – diameter of the shaft

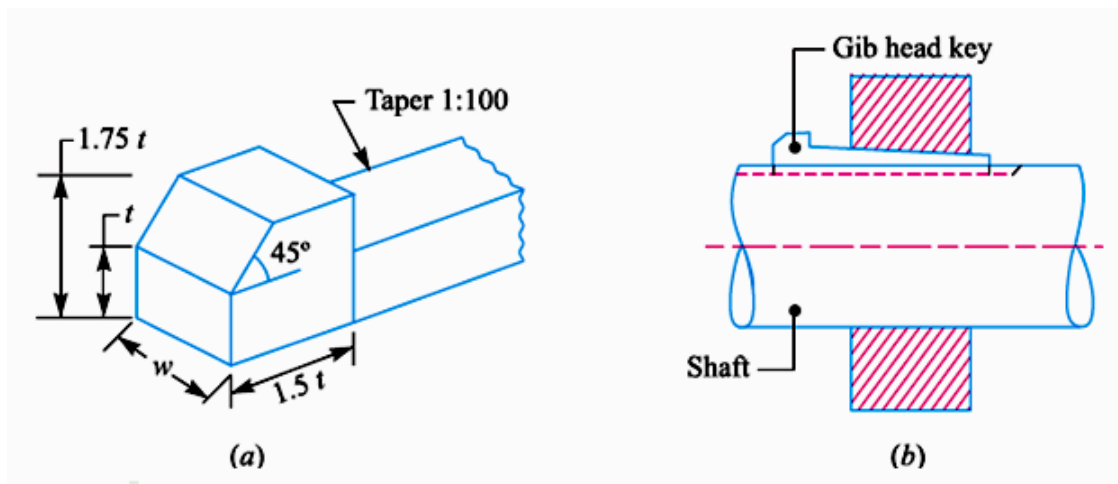
the key has taper 1 in 100 on the top side only

### **b. square sunk key**

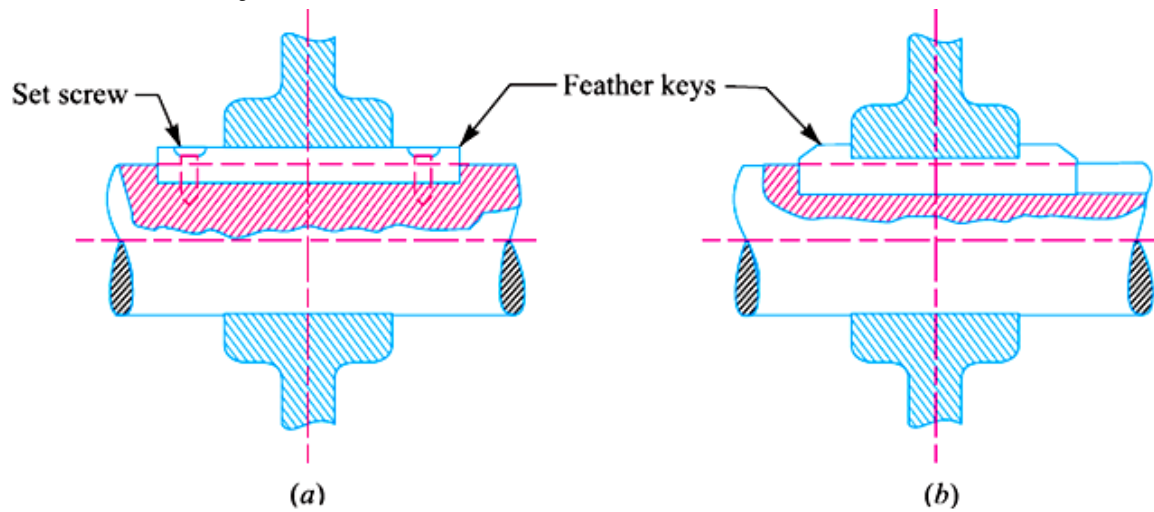
$$w = t = \frac{d}{4}$$

**c. parallel sunk key** : it is the same of others above just a taper less

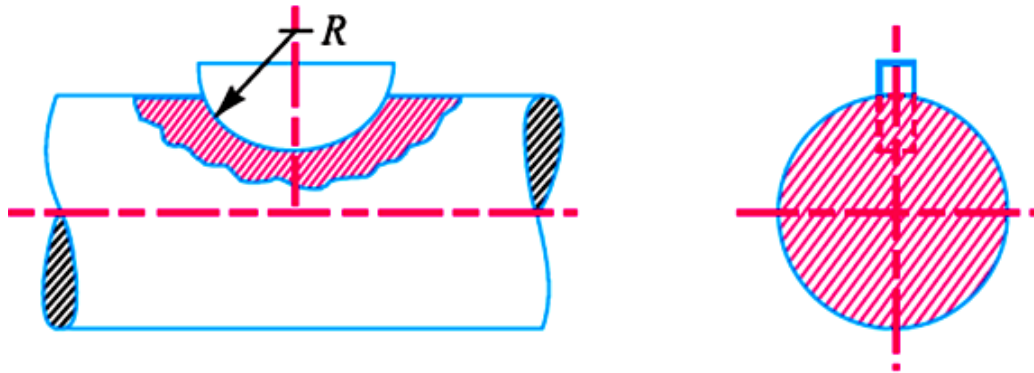
### **d. Gib head key**



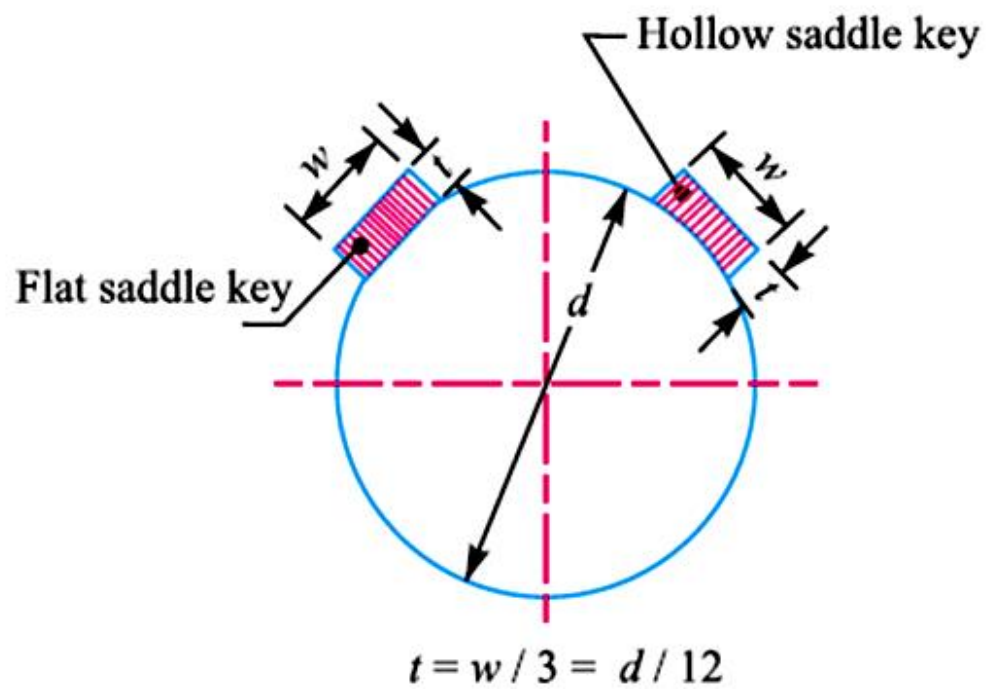
**e. feather key :**



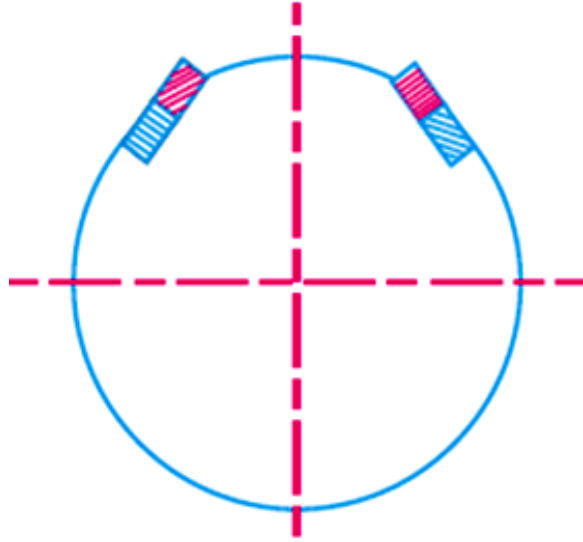
**f. woodruff key :**



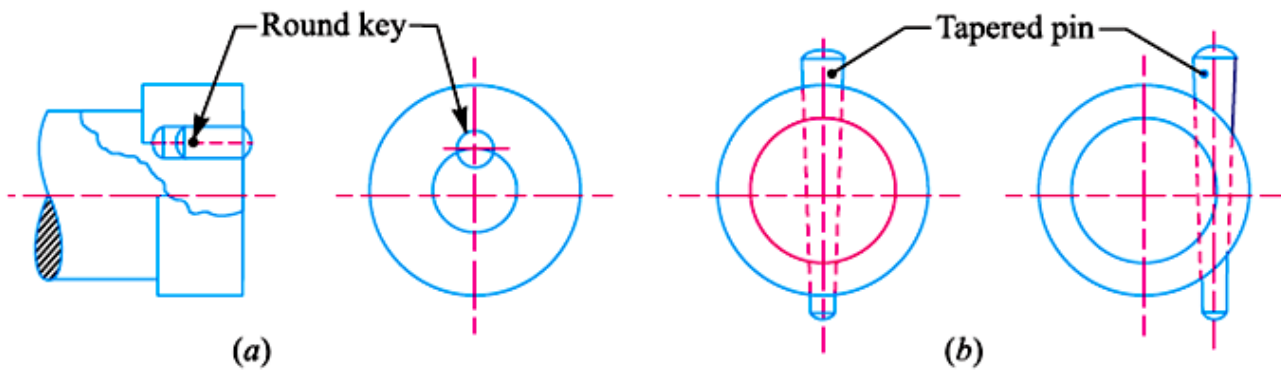
## saddle keys



## tangent keys

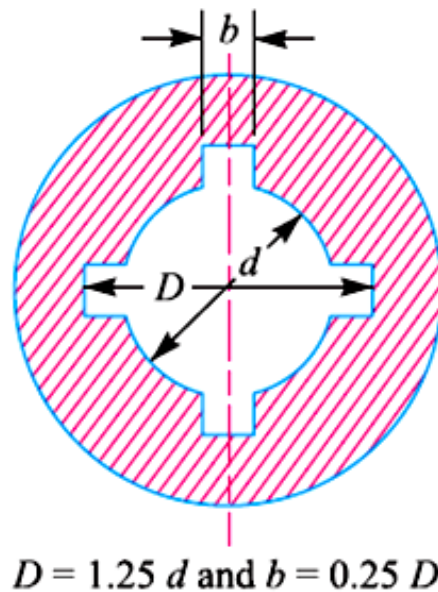


## round keys



## splines

these shafts usually have four ,six ,ten ,sixteen splines. the splined shafts are used when the force to be transmitted large in proportion to the size of the shaft.



### Stresses in keys

#### 1.shear stress

$$\tau = \frac{F}{A_s} = \frac{\frac{T}{r}}{l \times w} = \frac{\frac{T}{\frac{d}{2}}}{l \times w} = \frac{2T}{d \times l \times w}$$

$A_s$  - Area resisting shear

$T$  - transmitted torque

$w$  - width of key

$l$  - length of the key

$r$  – radius of shaft

## 2. Crushing stress

$$S_c = \frac{F}{A_c} = \frac{\frac{T}{r}}{\frac{t}{2} \times l} = \frac{\frac{T}{\frac{d}{2}}}{\frac{t \times l}{2}} = \frac{4T}{d \times t \times l}$$

$t$  - thickness of key

$A_c$  - area resisting crushing

$S_c$  - crushing stress

## Post Test

It is required to transmit 75 Hp from shaft 6 cm in diameter to a gear by a sunk key of length 70 mm at 1000 R.P.M permissible shear stress is  $60 \text{ MN/m}^2$  . and the crushing stress is  $120 \text{ MN/m}^2$  . Find the dimension of the key .

## Key Answer

## Pre Test

$$\tau = \frac{2T}{d \times l \times w}$$

$$= \frac{2 \times 14000}{4 \times 5 \times 1.4} = 1000 \text{ kg/cm}^2$$

$$S_c = \frac{4T}{d \times t \times l} = \frac{4 \times 14000}{4 \times 2 \times 5} = 1400 \text{ kg/cm}^2$$

1400 for crushing < 1680 kg/cm<sup>2</sup> ∴ 50 is enough ∴

∴ 1000 for shear > 560 kg/cm<sup>2</sup> ∴ 50 is not enough

To find require length of the key we need the shear law

$$\tau = \frac{2T}{d \times l \times w}$$

$$560 = \frac{2 \times 14000}{4 \times l \times 1.4}$$

$$l = 8.928 \text{ cm} = 89.28 \text{ mm}$$

∴ the length of key is about 90 mm

## Post Test

$$P = F \times V$$

P – power

F – force



V – velocity

$$T = F \times r = F \times \frac{d}{2} \Rightarrow \Rightarrow \Rightarrow F = \frac{2T}{d}$$

$$V = \frac{\pi \times d \times N}{60 \times 1000}$$

$$P = \frac{2T}{d} \times \frac{\pi \times d \times N}{60 \times 1000} = \frac{2\pi \times N \times T}{60 \times 1000}$$

$$75 \times 0.75 = \frac{2\pi \times 1000 \times T}{60 \times 1000}$$

$$T = 537 N.m = 53700 N.cm$$

$$\tau = \frac{2T}{d \times l \times w}$$

$$\frac{60 \times 10^6}{10^4} = \frac{2 \times 53700}{6 \times 7 \times w}$$

$$w = 0.426 cm = 4.26 mm$$

$$S_c = \frac{4T}{d \times t \times l}$$

$$\frac{120 \times 10^6}{10^4} = \frac{4 \times 53700}{6 \times t \times 7}$$

$$t = 0.426 cm = 4.26 mm$$

## Reference

R. S. Khurmi, J. K. Gupta, "Theory of machine"

**Foundation of Technical Education**

**Al-Dour Technical Institute**

**Mechanical Department**

**2<sup>nd</sup> Stage**

**Training Package**

**In**

# **Frictional Clutches, Type of Frictional Clutches , Design of Frictional Clutches**

**For**

**Students of second class**

**Mechanical Department/ Production**

**By**

**Nadum I. Naser**



**Overview**

**Clutches** : clutch is a machine member used to connect a driving shaft to a driven shaft , so that the driven shaft may be started or stopped at will, without stopping the drive shaft.

### **Objectives :-**

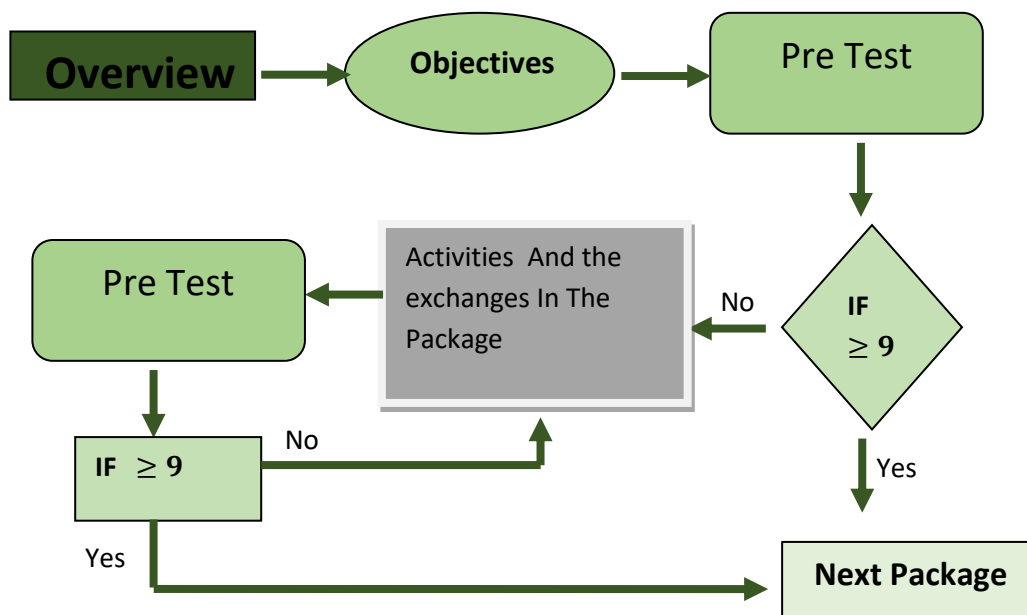
After studying the first modular unit , the student will be able to:-

1-Define **Types of clutches**

2- **Design of a disc clutch.**

3- Determine the maximum, minimum and the average pressure in a plate clutch.

### **Flow Chart:-**



### **Pre Test**

Determine the maximum, minimum and the average pressure in a plate clutch when the axial force is  $4\text{ KN}$ . The inside radius of the contact surface is  $50\text{ mm}$ , and the outside radius is  $100\text{ mm}$ , assume uniform wear?

## Notes

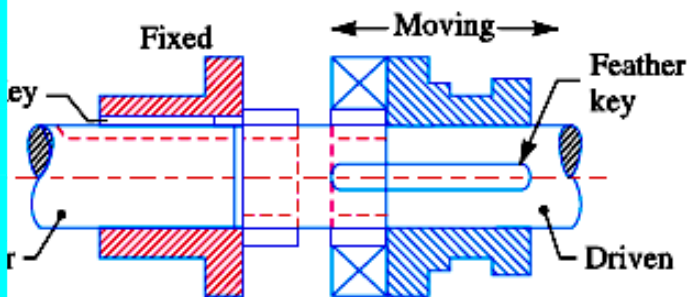
-10 degrees for the above question.

- Check your answers in key answer.

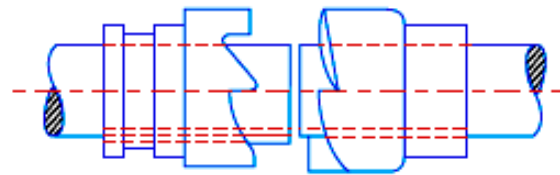
## The Text

There are two main types of clutches:

### 1. positive clutches



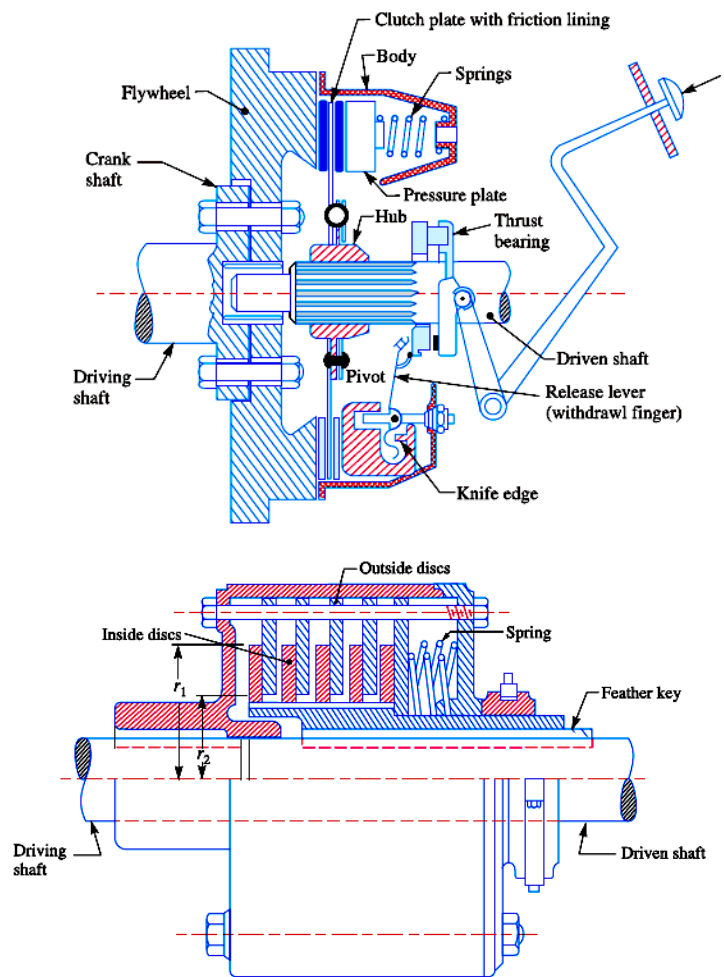
(a) Square jaw clutch.



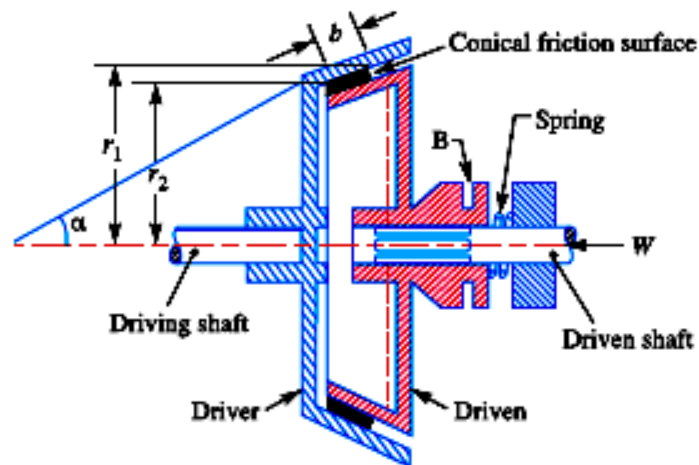
(b) Spiral jaw clutch.

### 2. friction clutches

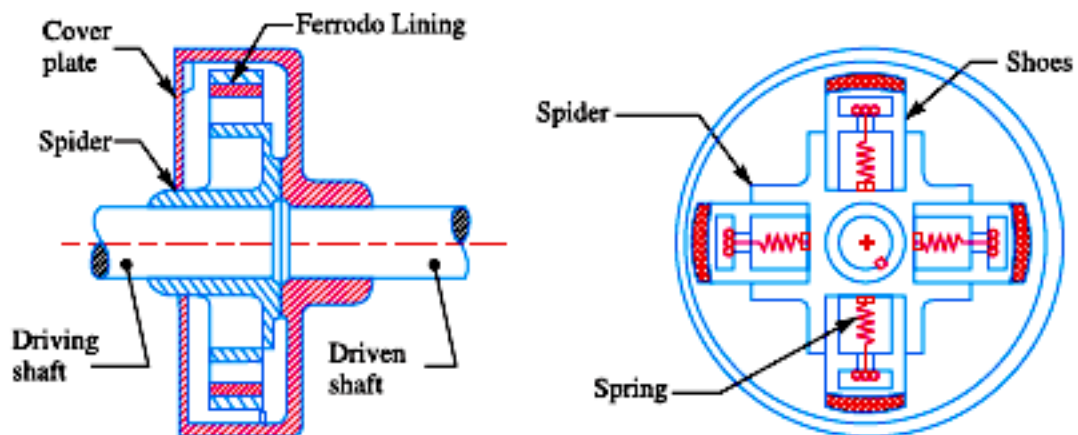
(a) disc or plate clutches



(b) cone clutches



(c) centrifugal clutches



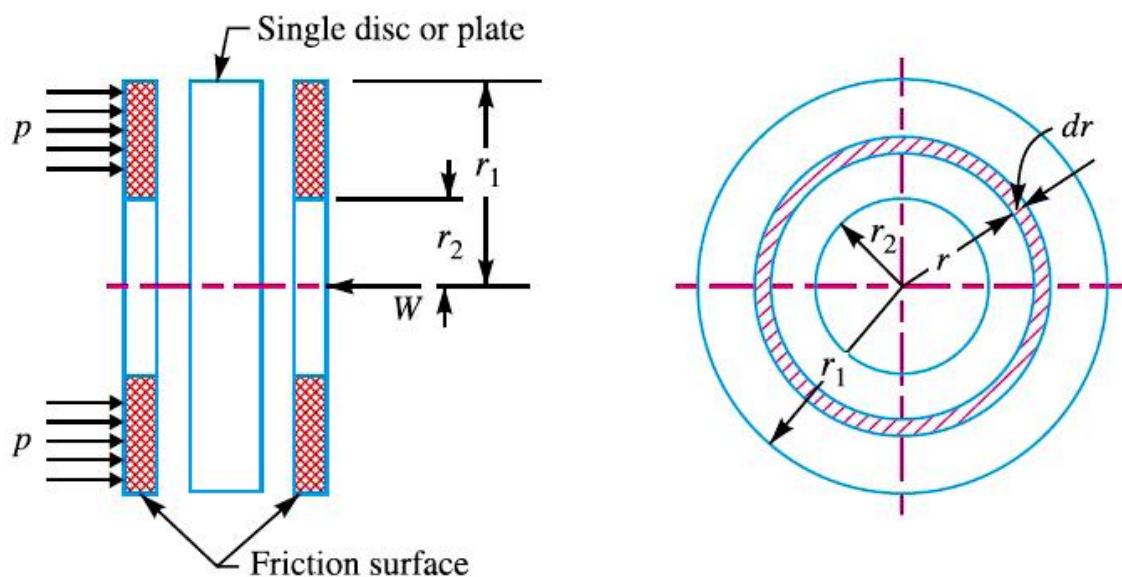
Note: the disc and cone clutches are known as ***axial friction clutches***, while the centrifugal clutch is called ***radial friction clutch***.



### Properties of materials commonly used for lining of friction surfaces.

Material of friction surfaces	Operating condition	Coefficient of friction	Maximum operating temperature ( $^{\circ}\text{C}$ )	Maximum pressure ( $\text{N/mm}^2$ )
iron on cast iron or steel	dry	0.15 – 0.20	250 – 300	0.25– 0.4
iron on cast iron or steel	In oil	0.06	250 – 300	0.6 – 0.8
tempered steel on Hardened steel	In oil	0.08	250	0.8 – 0.8
brass on cast iron or steel	In oil	0.05	150	0.4
pressed asbestos on cast iron or steel	dry	0.3	150 – 250	0.2 – 0.3
brass on cast iron or steel	dry	0.4	550	0.3
brass on cast iron or steel	In oil	0.1	550	0.8

### Design of a disc clutch:



Consider two friction surfaces maintained in contact by an axial thrust (  $W$  ) :



$T$  – torque transmitted by the clutch

$P$  – axial pressure with which the contact surfaces are held together

$r_1$  and  $r_2$  – external and internal radii of friction faces

consider an elementary ring of radius  $r$  and thickness  $dr$  :

then the area of contact surface  $A = 2\pi r.dr$

axial force on the ring  $\therefore$

$$\delta W = \text{pressure} \times \text{Area} = p \times A = p \times 2\pi r.dr$$

And the frictional force on the ring acting tangentially at radius  $r$  :

$$F_r = \mu \times \delta W = \mu.p \times 2\pi r.dr$$

$\mu$  – coefficient of friction

frictional torque acting on the ring:

$$T_r = F_r \times r = \mu.p \times 2\pi r.dr \times r = 2\pi \mu.p.r^2 dr$$

We shall considering two cases :

1.uniform pressure

2.uniform axial wear

**1.when we have uniform pressure**

$$p = \frac{W}{\pi[(r_1)^2 - (r_2)^2]}$$

$$T = \mu . W . R$$

$$R = \frac{2}{3} \left[ \frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2} \right]$$

$W$  – axial thrust on the friction surfaces

$T$  – total friction torque

$R$  – mean radius of the friction surface

## 2. when we have uniform wear

The normal wear is proportional to the work of friction. The work of friction is proportional to the product of normal pressure ( $p$ ) and the sliding velocity ( $V$ ):

normal wear    work of friction  $\propto p . V \propto$

$$- i - \quad p . V = K \quad (K - constant)$$

$$p = \frac{K}{V}$$

$$p \times r = C \quad (C - constant) \quad - ii -$$

$$p = \frac{C}{r}$$

$r$  – distance from the axis of clutch

$$\delta W = \text{pressure} \times \text{Area} = p \times A = p \times 2\pi r . dr$$

$$\delta W = \frac{C}{r} \times 2\pi r . dr = 2\pi C . dr$$

$$\therefore W = 2\pi C (r_1 - r_2)$$

$$C = \frac{W}{2\pi (r_1 - r_2)}$$

$$T_r = F_r \times r = \mu . p \times 2\pi r . dr \times r = 2\pi \mu . p . r^2 dr$$

$$T_r = 2\pi \mu \times \frac{C}{r} \times r^2 dr = 2\pi . \mu . C . r dr$$

$$\therefore T = \pi \mu C \left[ (r_1)^2 - (r_2)^2 \right] = \pi \mu \times \frac{W}{2\pi (r_1 - r_2)} \times \left[ (r_1)^2 - (r_2)^2 \right]$$

$$= \frac{1}{2} \mu W (r_1 + r_2) = \mu . W . R$$

$$R = \frac{r_1 + r_2}{2}$$

tes : In general, total frictional torque acting on the friction surfaces (or on the clutch) is given by

$$T = n \cdot \mu \cdot W \cdot R \quad \text{where}$$

$n$  = Number of pairs of friction (or contact) surfaces, and

$R$  = Mean radius of friction surface

$$= \frac{2}{3} \left[ \frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2} \right] \quad \dots \text{ (For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \quad \dots \text{ (For uniform wear)}$$

2. In a single disc or plate clutch, normally both sides of the disc are effective. Therefore a single disc has two pairs of surfaces in contact (*i.e.*  $n = 2$ ).

3. If the intensity of pressure is maximum at the inner radius ( $r_2$ ) of the friction or contact surface, before equation (i) may be written as

$$p_{max} \times r_2 = C \quad \text{or} \quad p_{max} = C / r_2$$

4. If the intensity of pressure is minimum at the outer radius ( $r_1$ ) of the friction or contact surface, before equation (i) may be written as

$$p_{min} \times r_1 = C \quad \text{or} \quad p_{min} = C / r_1$$

5. The average pressure ( $p_{av}$ ) on the friction or contact surface is given by

$$p_{av} = \frac{\text{Total force on friction surface}}{\text{Cross-sectional area of friction surface}} = \frac{W}{\pi [(r_1)^2 - (r_2)^2]}$$

6. In case of a new clutch, the intensity of pressure is approximately uniform, but in an old clutch, the uniform wear theory is more approximate.

7. The uniform pressure theory gives a higher friction torque than the uniform wear theory. Therefore in case of friction clutches, uniform wear should be considered, unless otherwise stated.

## Post Test

A plate clutch having a single driving plate with contact surfaces on each side is required to transmit 110 KW at 1250 R.P.M . the outer diameter of the contact surfaces is to be 300 mm. the coefficient of friction is 0.4 .assuming a uniform pressure of 0.17 N/mm<sup>2</sup> . determine the inner diameter of the friction surfaces.

## Key Answer

### Pre Test

*maximum pressure*

$$P_{\max} \times r_2 = C$$

$$P_{\max} \times 50 = C$$

$$\therefore C = 50 P_{\max}$$

$$W = 2\pi C (r_1 - r_2)$$

$$4 \times 10^3 = 2\pi \times 50 P_{\max} \times (100 - 50) = 15710 P_{\max}$$

$$P_{\max} = \frac{4000}{15710} = 0.2546 \text{ N / mm}^2$$

*minimum pressure*

$$P_{\min} \times r_1 = C$$

$$P_{\min} \times 100 = C$$

$$\therefore C = 100 P_{\min}$$

$$W = 2\pi C (r_1 - r_2)$$

$$4 \times 10^3 = 2\pi \times 100 P_{\min} \times (100 - 50) = 31420 P_{\min}$$

$$\therefore P_{\min} = \frac{4000}{31420} = 0.1273 \text{ N / mm}^2$$

*average pressure*

$$P_{av} = \frac{\text{total force on friction surface}}{\text{Cross-sectional area of friction surface}}$$

$$= \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{4000}{\pi[(100)^2 - (50)^2]}$$

$$P_{av} = 0.17 \text{ N / mm}^2$$

## Post Test

$$T = \frac{P \times 60}{2\pi N} = \frac{110 \times 10^3 \times 60}{2\pi \times 1250} = 840 \text{ N.m} = 840 \times 10^3 \text{ N.mm}$$

$$T = n \cdot \mu \cdot W \cdot R \quad -i-$$

$$n = 2$$

$$\mu = 0.4$$

$$W = \text{pressure} \times \text{area} = p \times \pi[(r_1)^2 - (r_2)^2] \\ = 0.17 \times \pi[(150)^2 - (r_2)^2] = 0.534[(150)^2 - (r_2)^2]$$

$$R = \frac{2}{3} \left[ \frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2} \right] = \frac{2}{3} \left[ \frac{(150)^3 - (r_2)^3}{(150)^2 - (r_2)^2} \right]$$

in - i-

$$840 \times 10^3 = 2 \times 0.4 \times 0.534[(150)^2 - (r_2)^2] \times \frac{2}{3} \left[ \frac{(150)^3 - (r_2)^3}{(150)^2 - (r_2)^2} \right] = 0.285[(150)^3 - (r_2)^3]$$

$$(150)^3 - (r_2)^3 = \frac{840 \times 10^3}{0.285} = 2.95 \times 10^6$$

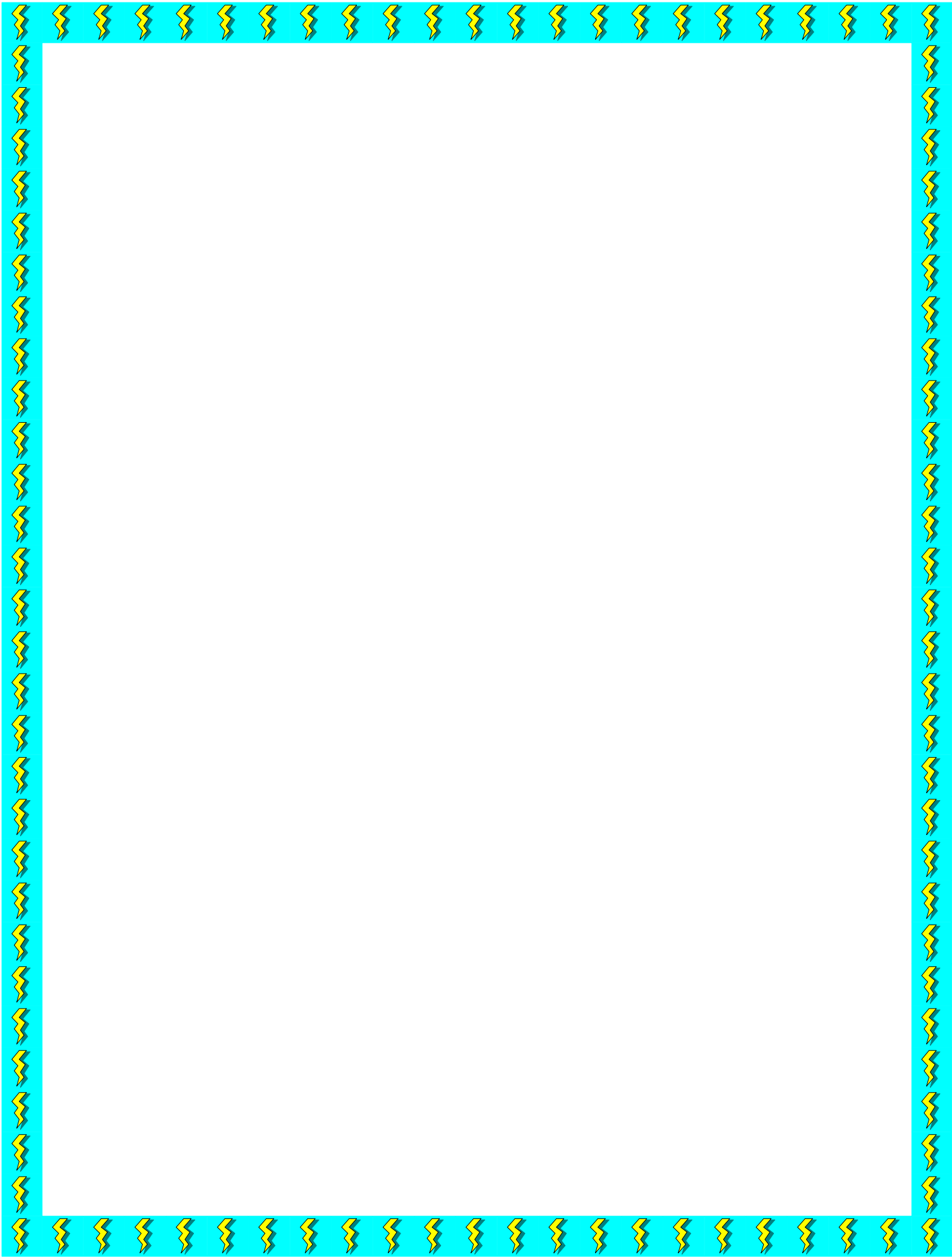
$$r_2^3 = 150^3 - 2.95 \times 10^6 = 0.425 \times 10^6$$

$$\therefore r_2 = 75 \text{ mm}$$

$$d_2 = 2 \times r_2 = 2 \times 75 = 150 \text{ mm}$$

## Reference

R. S. Khurmi, J. K. Gupta, "Theory of machine"



**Foundation of Technical Education**

**Al-Dour Technical Institute**

**Mechanical Department**

**2<sup>nd</sup> Stage**

**Training Package**

**In**

# **Types of Springs , Design of Springs**

**For**

**Students of second class**

**Mechanical Department/ Production**

**By**

**Nadum I. Naser**





## Overview

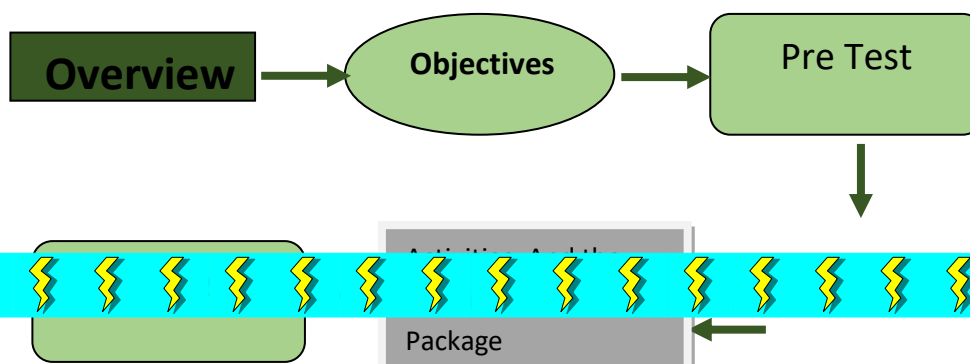
Spring is an elastic body compressed or extended when loaded , and go to its original shape when load is removed.

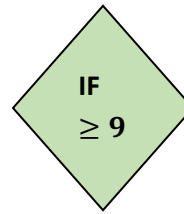
### Objectives :-

After studying the first modular unit , the student will be able to:-

- 1-Define Types of springs
- 2- Calculate stress due to curvature of wire.
- 3- Calculate torsional stress in the spring.

### Flow Chart:-





Next Package

## Pre Test

*A compression coil spring made of an alloy steel is having the following specifications:*

*mean diameter of coil = 50 mm; wire diameter = 5 mm; number of active coils = 20;*

*if this spring is subjected to an axial load of 500 N ; calculate the maximum shear stress (neglect the curvature effect) to which the spring material is subjected.*

## Notes

-10 degrees for the above question.

- Check your answers in key answer.

## The Text

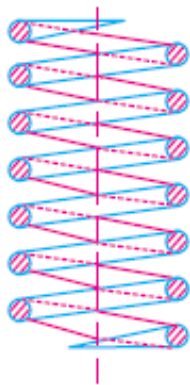
**Spring is an elastic body compressed or extended when loaded , and go to its original shape when load is removed.**

The important applications of springs are as follows :

- 1.to apply forces, as in brakes, clutches and spring loaded valves
- 2.to control motion by maintaining contact between two elements as in cams and followers.
- 3.to measure forces, as in spring balances and engine indicators
- 4.to store energy, as in watches, toys, etc..
- 5.to absorb or control energy due to either shock or vibration, as in car spring, aircraft landing gears, shock absorbers and vibration dampers.

## **Types of springs**

### **1.helical springs**

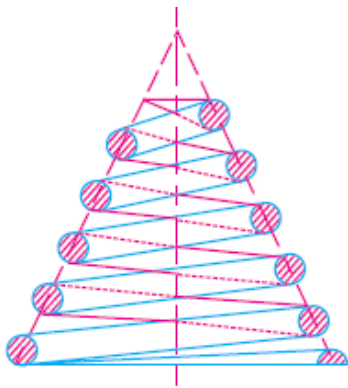


*(a)* Compression helical spring.

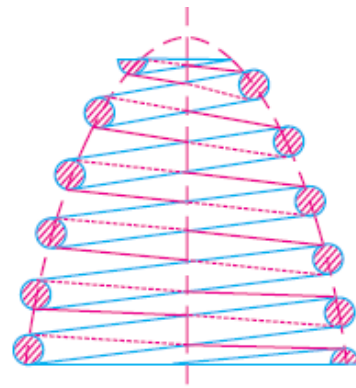


*(b)* Tension helical spring.

## 2.conical and volute springs

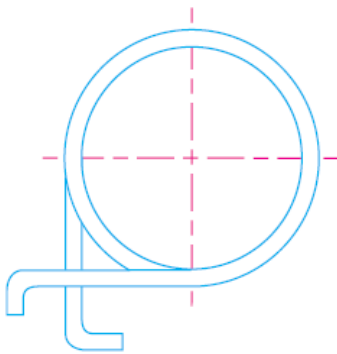


(a) Conical spring.

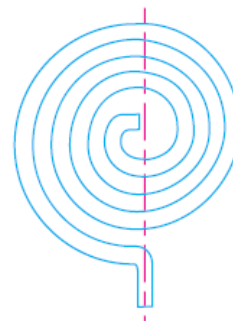
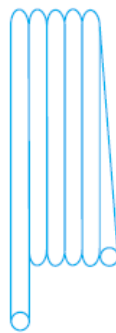


(b) Volute spring.

## 3.torsion springs

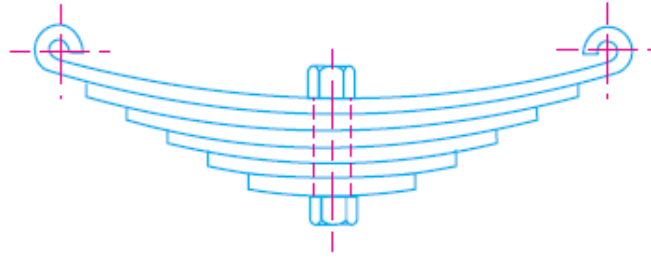


(a) Helical torsion spring.

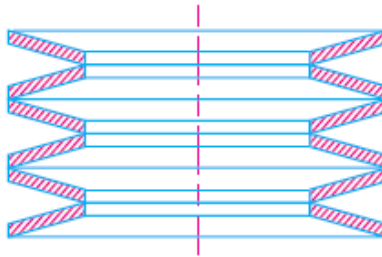


(b) Spiral torsion spring.

## 4.laminated or leaf springs



## 5. disc springs



## Terms used in compression springs

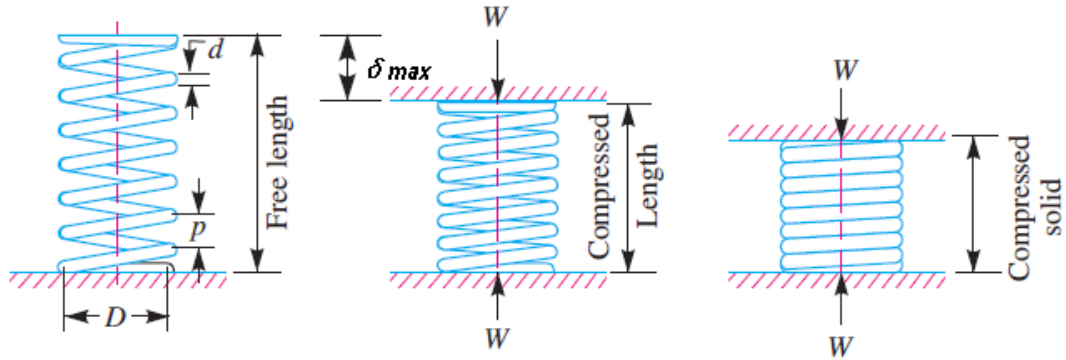
**1. solid length:** it's the product of total numbers of coils and the diameter of the wire

$$L_s = n' \times d$$

$n'$  – total number of coils

$d$  – diameter of wire

**2. free length :** it's the length of spring in the free or unloaded condition.



Free length = solid length + max. compression + clearance between adjacent coils

$$L_F = n' \times d + \delta_{\max} + 0.15 \delta_{\max}$$

We can also use the following relation to find the free length of spring

$$L_F = n' \times d + \delta_{\max} + (n' - 1) \times 1 \text{ mm}$$

The clearance is taken as 1 mm

**3. spring index :** it's the ratio of the mean diameter of the coil to the mean diameter of wire

$$C = \frac{D}{d}$$

**4. spring rate :** it's the load required per unit deflection of the spring

$$k = \frac{W}{\delta}$$

**5. pitch :** it means the axial distance between adjacent coils in uncompressed state.

$$P = \frac{\text{free length}}{n' - 1} = \frac{L_F}{n' - 1}$$

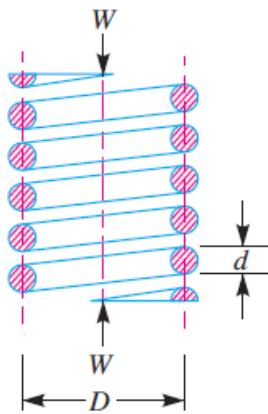
The pitch may also be obtained by using the following relation

$$P = \frac{L_F - L_S}{n} + d$$

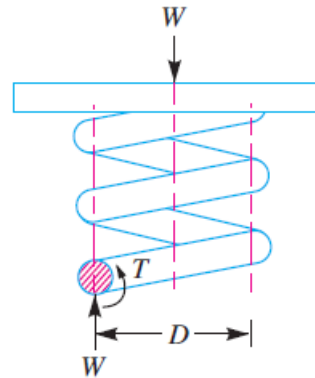
$L_F$  – free length

$L_S$  – solid length

## Stresses in helical springs of circular wire



(a) Axially loaded helical spring.



(b) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

$$T = W \times \frac{D}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\tau_1 = \frac{8 W D}{\pi d^3}$$

$\tau_1$  – torsional shear stress

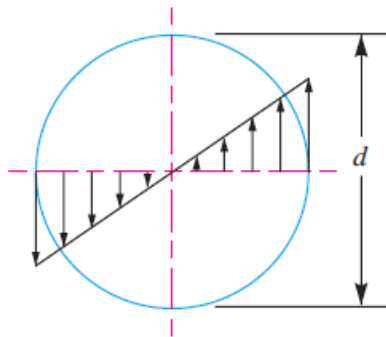
The following stresses also act on the wire

**1. direct shear stress because of the load W**

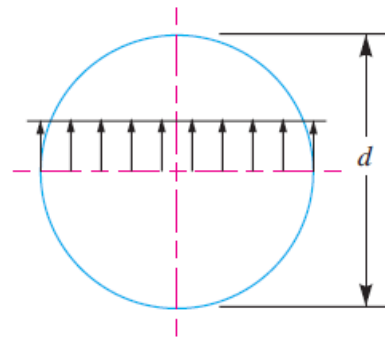
$$\tau_2 = \frac{\text{load}}{\text{cross-sectional area of the wire}}$$

$$\tau_2 = \frac{W}{\frac{\pi}{4} \times d^2} = \frac{4W}{\pi d^2}$$

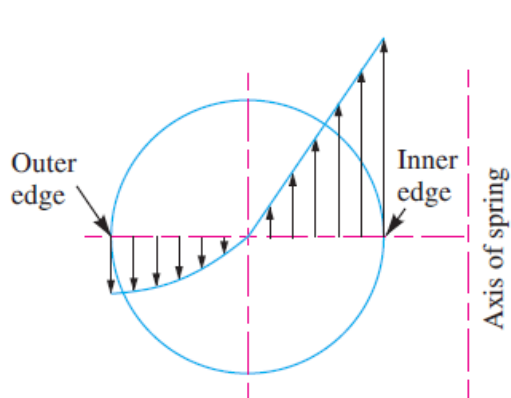
## 2. stress due to curvature of wire



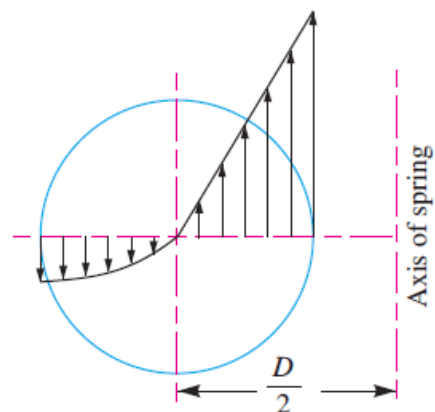
(a) Torsional shear stress diagram.



(b) Direct shear stress diagram.



(c) Resultant torsional shear and direct shear stress diagram.



(d) Resultant torsional shear, direct shear and curvature shear stress diagram.



Maximum shear stress in the wire = torsional shear stress + direct shear stress

$$\tau_{\max} = \tau_1 + \tau_2$$

$$= \frac{8 W D}{\pi d^3} + \frac{4 W}{\pi d^2} = \frac{8 W D}{\pi d^3} \left(1 + \frac{d}{2D}\right) \quad \text{if } \frac{D}{d} = C$$

$$\tau_{\max} = \frac{8 W D}{\pi d^3} \left(1 + \frac{1}{2C}\right) = K_s \times \frac{8 W D}{\pi d^3}$$

$$\text{where } K_s = 1 + \frac{1}{2C} \quad \text{shear stress factor}$$

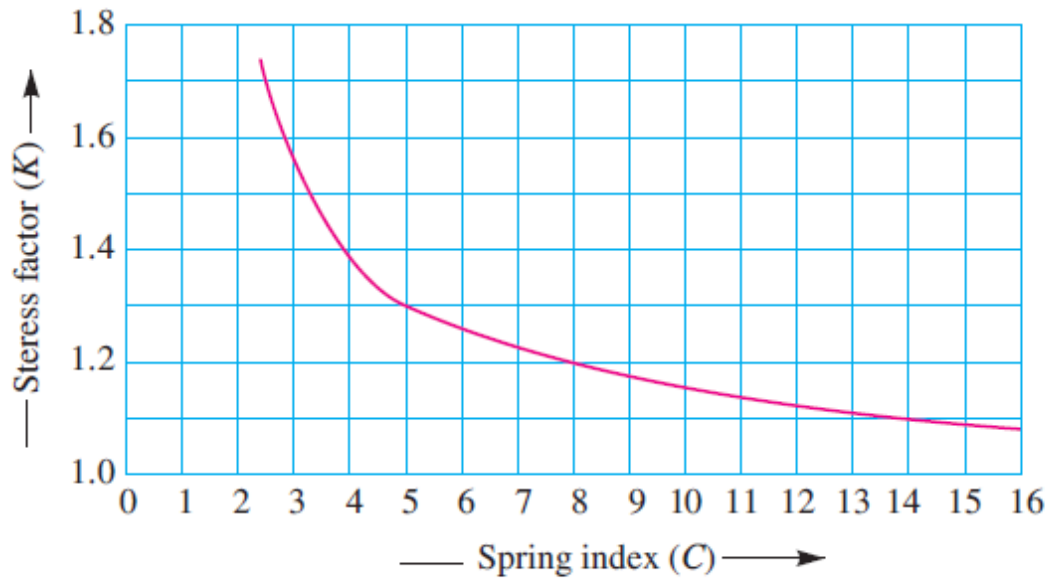
***In order to consider the effects of both direct shear stress as well as curvature of the wire we use a Wahl's stress factor.***

Maximum shear stress in the wire

$$\tau = K \times \frac{8 W D}{\pi d^3} = K \times \frac{8 W C}{\pi d^2}$$

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$$

*Wahl's stress factor*



Wahl's stress factor for helical springs.

## Deflection of helical springs of circular wire

$$\delta = \theta \times \frac{D}{2}$$

$\theta$  – Angular deflection of wire

$$\theta = \frac{16 W D^2 n}{G d^4}$$

$$\delta = \frac{16 W D^2 n}{G d^4} \times \frac{D}{2} = \frac{8 W D^3 n}{G d^4} = \frac{8 W C^3 n}{G d}$$

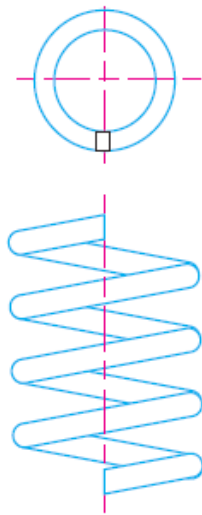
$n$  – number of active turn

$G$  – modulus of rigidity for the material of spring wire

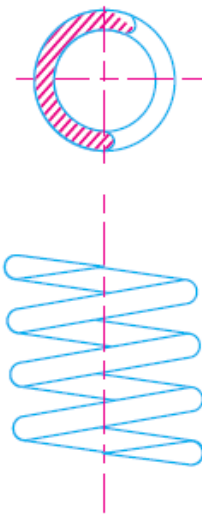
$$\frac{\delta}{n} = \frac{8 W D^3}{G d^4} = \frac{8 W C^3}{G d} \quad \text{deflection per active turn}$$

## Spring rate or stiffness of the spring

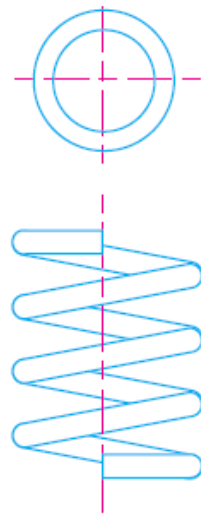
$$\frac{W}{\delta} = \frac{G d^4}{8 D^3 n} = \frac{G d}{8 C^3 n} = \text{constant}$$



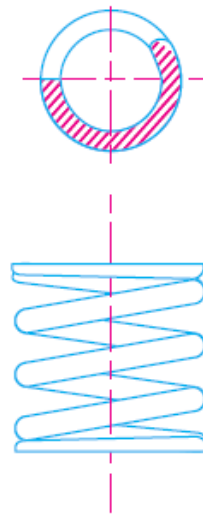
(a) Plain ends.



(b) Ground ends.



(c) Squared ends.



(d) Squared and ground ends.

Type of end	Total number of turns ( $n'$ )	Solid length	Free length
Plain ends	$n$	$(n + 1) d$	$p \times n + d$
Ground ends	$n$	$n \times d$	$p \times n$
Squared ends	$n + 2$	$(n + 3) d$	$p \times n + 3d$
Squared and ground ends	$n + 2$	$(n + 2) d$	$p \times n + 2d$

$n$  = Number of active turns,  
 $p$  = Pitch of the coils, and  
 $d$  = Diameter of the spring wire.

SWG	Diameter (mm)	SWG	Diameter (mm)	SWG	Diameter (mm)	SWG	Diameter (mm)
7/0	12.70	7	4.470	20	0.914	33	0.2540
6/0	11.785	8	4.064	21	0.813	34	0.2337
5/0	10.973	9	3.658	22	0.711	35	0.2134
4/0	10.160	10	3.251	23	0.610	36	0.1930
3/0	9.490	11	2.946	24	0.559	37	0.1727
2/0	8.839	12	2.642	25	0.508	38	0.1524
0	8.229	13	2.337	26	0.457	39	0.1321
1	7.620	14	2.032	27	0.4166	40	0.1219
2	7.010	15	1.829	28	0.3759	41	0.1118
3	6.401	16	1.626	29	0.3454	42	0.1016
4	5.893	17	1.422	30	0.3150	43	0.0914
5	5.385	18	1.219	31	0.2946	44	0.0813
6	4.877	19	1.016	32	0.2743	45	0.0711

## Post Test

A helical spring is made from a wire of 6 mm diameter and has a diameter of 70 mm; if the permissible shear stress is 350 MPa and modulus of rigidity 84 KN/mm<sup>2</sup>. find the axial load which spring can carry and the deflection per active turn.

## Key Answer

## Pre Test

Given :  $D = 50 \text{ mm}$  ;  $d = 5 \text{ mm}$  ;  $*n = 20$  ;  $W = 500 \text{ N}$

We know that the spring index,

$$C = \frac{D}{d} = \frac{50}{5} = 10$$

∴ Shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 10} = 1.05$$

and maximum shear stress (neglecting the effect of wire curvature),

$$\tau = K_S \times \frac{8W.D}{\pi d^3} = 1.05 \times \frac{8 \times 500 \times 50}{\pi \times 5^3} = 534.7 \text{ N/mm}^2$$

## **Post Test**

$$C = \frac{D}{d} = \frac{70}{6} = 11.6$$

**1. neglecting the effect of curvature**

$$K_s = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 11.6} = 1.312$$

$$\tau_{\max} = K_s \times \frac{8 W D}{\pi d^3}$$

$$350 = 1.312 \times \frac{8 \times W \times 70}{\pi \times 6^3} = 1.082 W$$

$$W = \frac{350}{1.082} = 323.5 \text{ N}$$

$$\delta = \frac{8 W D^3 n}{G d^4}$$

$$\therefore \frac{\delta}{n} = \frac{8 W D^3}{G d^4} = \frac{8 \times 323.5 \times 70^3}{84 \times 10^3 \times 6^4} = 8.15 \text{ mm}$$

**2. considering the effect of curvature**

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 11.6 - 1}{4 \times 11.6 - 4} + \frac{0.615}{11.6} = 1.124$$

$$\tau = K \times \frac{8 W C}{\pi d^2}$$

$$350 = 1.124 \times \frac{8 \times W \times 11.6}{\pi \times 6^2} = 0.922 W$$

$$W = \frac{350}{0.922} = 379.6 \text{ N}$$

$$\delta = \frac{8 W D^3 n}{G d^4}$$

$$\frac{\delta}{n} = \frac{8 \times 379.6 \times 70^3}{84 \times 10^3 \times 6^4} = 9.56 \text{ mm}$$

## Reference

R. S. Khurmi, J. K. Gupta, "Theory of machine"